Print your name **neatly**:

Last name: ____________________________________________________________

First name: __________________________________________________________

Sign your name: _______________________________________________________

Please fill in your Student ID number (UIN): __ __ __ __ __ __ __ __ __

**IMPORTANT**

Read these directions carefully:

- There are 7 problems totalling 100 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.

- **Indicate what you are doing!** We cannot give full credit for merely writing down the answer. **Neatness counts!** I will give generous partial credit if I can tell that you are on the right track. This means you must be *neat* and organized.

- Each problem with its associated figure is self explanatory. If you *must* ask a question, then come to the front, being as discrete as possible so as not to disturb others.

- Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.
Problem 1. 10 points.

Two small charges, $-Q$ and $+Q$, are separated by a distance $2d$ where $d$ is a given positive constant. Calculate the electric field a perpendicular distance $y$ from the midpoint of the line connecting the charges (this point is shown in the Figure).

Use the coordinate system shown. Remember that electric field is a vector, so calculate both the $x$ and $y$ components of the field.

Express your answer in terms of $Q$, $y$, $d$, physical constants such as $\epsilon_0$, and numerical factors.
Two thin rods, each of length \( d \), carry charges \(-Q\) and \(+Q\) distributed uniformly along their length. The two rods are abutted end to end as shown in the Figure. Calculate the electric field a perpendicular distance \( y \) from the midpoint of the line connecting the charges (this point is shown in the Figure).

Use the coordinate system shown. Remember that electric field is a vector, so calculate both the \( x \) and \( y \) components of the field.

Express your answer in terms of \( Q \), \( y \), \( d \), physical constants such as \( \varepsilon_0 \), and numerical factors.

If you work \textbf{neatly} I will find more partial credit for you!
Problem 3. 10 points.

Three small spheres shown below carry charges $q_1$, $q_2$, and $q_3$ respectively. Find the net electric flux through each of the five closed surfaces shown in cross section in the Figure.

Express your answers in terms of $q_1$, $q_2$, $q_3$, physical constants such as $\epsilon_0$, and numerical factors.

(a) flux through Surface S1 =

(b) flux through Surface S2 =

(c) flux through Surface S3 =

(d) flux through Surface S4 =

(e) flux through Surface S5 =

Make sure you are being neat. Working neatly will help you get it right.
Problem 4. (20 points)

A conducting sphere of radius $R_1$ carries a net charge $-Q$.

Surrounding this sphere is a hollow spherical shell of nonconducting material. This shell has inner radius $R_2$ and outer radius $R_3$. A total charge $+Q$ is distributed uniformly throughout the spherical shell.

This is shown in cross section below.

Calculate the electric field in each of the following regions:

(a) points inside the conducting sphere: $r < R_1$.
(b) points between the conducting sphere and the insulating shell: $R_1 < r < R_2$.
(c) points inside the insulating shell: $R_2 < r < R_3$.
(d) points outside the insulating shell: $r > R_3$.

If you need more room to work neatly, you can continue this problem on the next page.
Problem 5. (10 points)

Two large parallel conducting plates are separated by a distance $d$. A spring of force constant $k$ is fixed to one of the plates, and on its other end is a charge $Q$ as shown in the Figure. Ignore gravity in this problem.

Suppose that the electric potential difference between the plates is increased from 0 to $V$. This will cause the charge $Q$ to push down on the spring.

Calculate the equilibrium distance the charge pushes down the spring. Express your answer in terms of $Q$, $V$, $k$, and $d$.

Hint: Remember Hooke’s law for springs says that a force $F$ will displace the spring a distance $F/k$ from equilibrium.

Present your work neatly and clearly.
Problem 6. (20 points)

A rod is bent into a quarter-circle of radius $R$. The rod carries a charge $+Q$ uniformly distributed along its length.

Choose your coordinate system so that the center of the circle is at the coordinate origin and the rod takes up the upper left quadrant. This is shown in the Figure:

(a) Calculate the electric potential $V$ at the origin. Take the potential to be zero infinitely far away. Express your answer in terms of $Q$, $R$, physical constants such as $\epsilon_0$, and numerical factors.

(b) Suppose a small particle of mass $m$ and charge $q$ is released from rest at the center of curvature of the rod (that is, at the coordinate origin). When the particle is released, which way will it initially move?

(c) Calculate the speed of the particle after it has moved very far from the rod.
Suppose there is some charge distribution that produced an electric potential that depends only on the $x$ coordinate according to the formula:

$$V(x) = \frac{1}{4\pi \varepsilon_0} \frac{2q}{\sqrt{a^2 + x^2}}$$

where $a$ is some given positive constant and $q$ is a measure of the charge.

Calculate the electric field. Express your answer in terms of $q$, $a$, $\varepsilon_0$, numerical factors, and of course, $x$. 

Working neatly will help you think about what you are doing.
Derivatives:

\[
\frac{d}{dx} ax^n = an x^{n-1}
\]

\[
\frac{d}{dx} \sin ax = a \cos ax
\]

\[
\frac{d}{dx} \cos ax = -a \sin ax
\]

\[
\frac{d}{dx} e^{ax} = ae^{ax}
\]

\[
\frac{d}{dx} \ln ax = \frac{1}{x}
\]

Integrals:

\[
\int a x^n \, dx = a \frac{x^{n+1}}{n+1}
\]

\[
\int \frac{dx}{x} = \ln x
\]

\[
\int \sin ax \, dx = -\frac{1}{a} \cos ax
\]

\[
\int \cos ax \, dx = \frac{1}{a} \sin ax
\]

\[
\int e^{ax} \, dx = \frac{1}{a} e^{ax}
\]

\[
\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a}
\]

\[
\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left( \sqrt{x^2 + a^2} + x \right)
\]

\[
\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a}
\]

\[
\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2} \frac{x}{\sqrt{x^2 + a^2}}
\]

\[
\int \frac{x \, dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}
\]