Physics 208: Electricity and Magnetism.
Final Exam, Secs. 501–505 10 May 2006
Instructor: Dr. George R. Welch, 301D Doherty Bldg, 845-1571

Print your name **neatly**:

Last name: ______________________________
First name: ______________________________

*Sign* your name: ______________________________

Please fill in your Student ID number (UIN): ____________–__________–__________–__________

**IMPORTANT**

Read these directions carefully:

- There are 10 problems totalling 200 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.

- **Indicate what you are doing!** We cannot give full credit for merely writing down the answer. **Neatness counts!** I will give generous partial credit if I can tell that you are on the right track. This means you must be *neat* and organized.

- Each problem with its associated figure is self explanatory. If you *must* ask a question, then come to the front, being as discrete as possible so as not to disturb others.

- Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.
Problem 1. 20 points.

Two charges, $+Q$ and $-2Q$, are separated by a distance $a$. Calculate the electric field at a distance $b$ above the negative charge. Use the $x$ and $y$ axes shown for your coordinate system. Calculate both components of the electric field.

Express your answers in terms of $Q$, $a$, $b$, physical constants, and numerical factors.
Problem 2. 25 points.

A rod of length $a$ carries a charge $Q$ uniformly distributed along its length. Calculate the electric potential at a perpendicular distance $b$ from one end of the rod.

Calculate
Potential
Here $\times$

$+Q$

$\leftarrow a \rightarrow$

If you work **neatly** I will find more partial credit for you!
Problem 3. 25 points.

A sphere of radius $R_1$ has a charge $+Q$ uniformly distributed throughout its volume. Surrounding this sphere and concentric with it is a thin spherical shell of radius $R_2$ which carries a charge $-Q$ uniformly distributed over its surface.

Calculate the electric field as a function of the distance $r$ from the center of the spheres for the following three cases:

(a) $r < R_1$

(b) $R_1 < r < R_2$

(c) $R_2 < r$

Express your answers in terms of $Q$, $R_1$, $R_2$, physical constants, and numerical factors.

NOTE: You must show your work and indicate what you are doing. I deliberately made this easy, so you MUST show the process, not just the answer.

Make sure you are being neat. Working neatly will help you get it right.
Problem 4. (20 points)

In the circuit shown below, there are two batteries and three resistors. The EMFs of the batteries and resistances of the resistors are labelled.

Calculate the current in each resistor. Make sure you make it clear which is which! For example, you could say which is right, center, and left. Similarly, you could call them 1, 2, and 3, but you must clearly indicate which is which. You must also make it clear which direction the current is flowing in each case.

**IMPORTANT:** You must write your equations neatly! I will not give full credit unless your equations and answers are neat and organized.
Problem 5. (15 points)

(a) Three resistors, each with the same resistance $R$, are connected in a network as shown. Calculate the equivalent resistance between points A and B. Express your answer in terms of $R$ and numerical factors.

(a) Three capacitors, each with the same capacitance $C$, are connected in a network as shown. Calculate the equivalent capacitance between points A and B. Express your answer in terms of $C$ and numerical factors.

Work neatly! If you are neat, I can read what you did and maybe find more points for you.
Problem 6. (20 points)

A long straight wire has a loop of radius $R$ in the middle. Current $I$ flows down the straight part of the wire, and then around the loop, and then down the the rest of the straight part. (This is what the dashed lines are supposed to indicate.)

Calculate the magnetic field at the center of the loop. Express your answer in terms of $I$, $R$, physical constants, and numerical factors.
Problem 7. (25 points)

It is possible to arrange current-carrying coils in such a way as to produce a magnetic field that varies approximately linearly with position. This type of arrangement is called “anti-Helmholz” coils and is fairly complicated.

Suppose instead we have a simplified arrangement where the magnetic field varies linearly in one dimension (which we will call $x$) and is constant in the perpendicular dimension. Let us further suppose that the coils that produce this field are driven by a sinusoidally alternating current, so that the magnetic field over some region of space can be written as:

$$B = B(x, t) = \frac{B_1}{c} x \cos \omega t$$

where $B_1$ is a constant that depends on how strong the field is, $c$ is a constant that determines how rapidly the field varies in space, and $\omega$ is the circular frequency with which the field varies in time.

Suppose we place a rectangular loop of conductor in a plane perpendicular to the magnetic field, with one edge of the loop at the position corresponding to $x = 0$ above. The rectangular loop has length $a$ in the $x$ direction, and length $b$ in the $y$ direction. Suppose that the total resistance of the conductive rectangular loop is $R$.

(a) Calculate the current induced in the loop. Express your answer in terms of the constants given above, and numerical factors. Your answer will be a function of time.

(b) Calculate the net force on the loop. Use this identity: $\cos \theta \sin \theta = \frac{1}{2} \sin 2\theta$. Your answer should be very interesting — the force on the loop oscillates back and forth twice as fast as the magnetic field that produces the force!

Please work this problem on the next page.

Working neatly will help you think about what you are doing.
Problem 8. (15 points)

Consider the circuit shown. The battery maintains a potential difference $V$, the resistors have equal resistance $R$, and the inductor has an inductance of $L$.

(a) Suppose the switch has been open a long time, and then the switch is closed. Calculate the current through each resistor immediately after the switch is closed.

(b) Calculate the current through each resistor a long time after the switch was closed in part (a).

(c) Suppose that after the switch had been closed a long time as in part (b), the switch is opened again. Calculate the current in each resistor immediately after the switch is opened.

(d) Calculate the current through each resistor a long time after the switch was opened in part (c).

Neatly explain your reasoning!
Problem 9. (15 points)

A high-power industrial welding laser produces a beam of light such that the maximum value of the magnetic field in the traveling EM-wave is $2.9 \times 10^{-3}$ Tesla. This may not sound like much, but it is nearly 100 times as big as the earth’s magnetic field.

(a) Calculate the average power output of this laser, supposing that the intensity is constant across the laser beam, with a 1 mm diameter.

(b) Calculate how much electromagnetic energy is contained in a 1 meter length of this laser beam.

You need a calculator for this – sorry.
Problem 10. (20 points)

(a) (6 points) An object is placed 4 cm in front of a concave spherical mirror. The focal length of the mirror is 5 cm. Calculate where the image is formed. Is it real or virtual? Is it erect or inverted?

(b) (6 points) A thin diverging lens has focal length \( f = -10 \text{ cm} \). An object is placed 10 cm from the lens. Calculate the position of the image of the object. Is it real or virtual? Is it erect or inverted?

(c) (8 points) A thin converging lens with focal length \( f = 5 \text{ cm} \) is placed 10 cm to the right of the lens described above. Calculate the position of the image produced by the second lens. Is it real or virtual? Is it erect or inverted?

Present your work neatly and clearly.