Physics 208 — Formula Sheet for Exam 1

You may remove this sheet. If you do remove this sheet, then do NOT turn it it!

**Force on a charge:**
An electric field $\vec{E}$ exerts a force $\vec{F}$ on a charge $q$ given by:

$$\vec{F} = q\vec{E}$$

**Coulomb’s law:**
A point charge $q$ located at the coordinate origin gives rise to an electric field $\vec{E}$ given by

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$

where $r$ is the distance from the origin (spherical coordinate), $\hat{r}$ is the spherical unit vector, and $\epsilon_0$ is the permittivity of free space:

$$\epsilon_0 = 8.8542 \times 10^{-12} \text{C}^2/(\text{N} \cdot \text{m}^2)$$

**Superposition:**
The principle of superposition of electric fields states that the electric field $\vec{E}$ of any combination of charges is the vector sum of the fields caused by the individual charges

$$\vec{E} = \sum_i \vec{E}_i$$

To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements and integrate all these elements:

$$\vec{E} = \int d\vec{E} = \int_q \frac{dq}{4\pi\epsilon_0 r^2} \hat{r}$$

**Electric flux:**
Electric flux is a measure of the “flow” of electric field through a surface. It is equal to the product of the area element and the perpendicular component of $\vec{E}$ integrated over a surface:

$$\Phi_E = \int E \cos \phi \, dA = \int \vec{E} \cdot \hat{n} \, dA = \int \vec{E} \cdot d\vec{A}$$

where $\phi$ is the angle from the electric field $\vec{E}$ to the surface normal $\hat{n}$.

**Gauss’ Law:**
Gauss’ law states that the total electric flux through any closed surface is determined by the charge enclosed by that surface:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

**Electric conductors:**
The electric field inside a conductor is zero. All excess charge on a conductor resides on the surface of that conductor.

**Electric Potential:**
The electric potential is defined as the potential energy per unit charge. If the electric potential at some point is $V$ then the electric potential energy at that point is $U = qV$. The electric potential function $V(\vec{r})$ is given by the line integral:

$$V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{l} + V(\vec{r}_0)$$

Beware of the minus sign. This gives the potential produced by a point charge $q$:

$$V = \frac{q}{4\pi\epsilon_0 r}$$

for a collection of charges $q_i$

$$V = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i}$$

and for a continuous distribution of charge

$$V = \int \frac{dq}{4\pi\epsilon_0 r}$$

where in each of these cases, the potential is taken to be zero infinitely far from the charges.

**Field from potential:**
If the electric potential function is known, the vector electric field can be derived from it:

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

or in vector form:

$$\vec{E} = -\left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)$$

Beware of the minus sign.