These are your formula sheets

DO NOT TURN IT IN!

Derivatives:

\[
\frac{d}{dx} ax^n = an x^{n-1}
\]

\[
\frac{d}{dx} \sin ax = a \cos ax
\]

\[
\frac{d}{dx} \cos ax = -a \sin ax
\]

\[
\frac{d}{dx} e^{ax} = ae^{ax}
\]

\[
\frac{d}{dx} \ln ax = \frac{1}{x}
\]

Integrals:

\[
\int ax^n \, dx = \frac{a x^{n+1}}{n + 1}
\]

\[
\int \frac{dx}{x} = \ln x
\]

\[
\int \sin ax \, dx = -\frac{1}{a} \cos ax
\]

\[
\int \cos ax \, dx = \frac{1}{a} \sin ax
\]

\[
\int e^{ax} \, dx = \frac{1}{a} e^{ax}
\]

\[
\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a}
\]

\[
\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left( \sqrt{x^2 + a^2} + x \right)
\]

\[
\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a}
\]

\[
\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2 \sqrt{x^2 + a^2}} x
\]

\[
\int \frac{x \, dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}
\]

DO NOT TURN THESE SHEETS IN!
Capacitance:
A capacitor is any pair of conductors separated by an insulating material. When the conductors have equal and opposite charges $Q$ and the potential difference between the two conductors is $V_{ab}$, then the definition of the capacitance of the two conductors is
\[ C = \frac{Q}{V_{ab}} \]
The energy stored in the electric field is
\[ U = \frac{1}{2} CV^2 \]
If the capacitor is made from parallel plates of area $A$ separated by a distance $d$, where the size of the plates is much greater than $d$, then the capacitance is given by
\[ C = \frac{\epsilon_0 A}{d} \]
Capacitors in series:
\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... \]
Capacitors in parallel:
\[ C_{eq} = C_1 + C_2 + ... \]
If a dielectric material is inserted, then the capacitance increases by a factor of $K$ where $K$ is the dielectric constant of the material
\[ C = KC_0 \]
Current:
When current flows in a conductor, we define the current as the rate at which charge passes:
\[ I = \frac{dQ}{dt} \]
We define the current density as the current per unit area, and can relate it to the drift velocity of charge carriers by
\[ \vec{J} = nq\vec{v}_d \]
where $n$ is the number density of charges and $q$ is the charge of one charge carrier.

Ohm’s Law and Resistance:
Ohm’s Law states that a current density $J$ in a material is proportional to the electric field $E$. The ratio $\rho = E/J$ is called the resistivity of the material. For a conductor with cylindrical cross section, with area $A$ and length $L$, the resistance $R$ of the conductor is
\[ R = \frac{\rho L}{A} \]
A current $I$ flowing through the resistor $R$ produces a potential difference $V$ given by
\[ V = IR \]
Resistors in series:
\[ R_{eq} = R_1 + R_2 + ... \]
Resistors in parallel:
\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... \]
Power:
The power transferred to a component in a circuit by a current $I$ is
\[ P = VI \]
where $V$ is the potential difference across the component.

Kirchhoff’s rules:
The algebraic sum of the currents into any junction must be zero:
\[ \sum I = 0 \]
The algebraic sum of the potential differences around any loop must be zero.
\[ \sum V = 0 \]

RC Circuits:
When a capacitor $C$ is charged by a battery with EMF given by $\mathcal{E}$ in series with a resistor $R$, the charge on the capacitor is
\[ q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right) \]
where $t = 0$ is when the the charging starts.
When a capacitor $C$ that is initially charged with charge $Q_0$ discharges through a resistor $R$, the charge on the capacitor is
\[ q(t) = Q_0e^{-t/RC} \]
where $t = 0$ is when the the discharging starts.
Force on a charge:
An electric field $\vec{E}$ exerts a force $\vec{F}$ on a charge $q$ given by:
$$\vec{F} = q\vec{E}$$

Coulomb’s law:
A point charge $q$ located at the coordinate origin gives rise to an electric field $\vec{E}$ given by
$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$
where $r$ is the distance from the origin (spherical coordinate), $\hat{r}$ is the spherical unit vector, and $\epsilon_0$ is the permittivity of free space:
$$\epsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$$

Superposition:
The principle of superposition of electric fields states that the electric field $\vec{E}$ of any combination of charges is the vector sum of the fields caused by the individual charges
$$\vec{E} = \sum_i \vec{E}_i$$
To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements and integrate all these elements:
$$\vec{E} = \int d\vec{E} = \int \frac{dq}{4\pi\epsilon_0 r^2} \hat{r}$$

Electric flux:
Electric flux is a measure of the “flow” of electric field through a surface. It is equal to the product of the area element and the perpendicular component of $\vec{E}$ integrated over a surface:
$$\Phi_E = \int E \cos \phi \, dA = \int \vec{E} \cdot \hat{n} \, dA = \int \vec{E} \cdot d\vec{A}$$
where $\phi$ is the angle from the electric field $\vec{E}$ to the surface normal $\hat{n}$.

Gauss’ Law:
Gauss’ law states that the total electric flux through any closed surface is determined by the charge enclosed by that surface:
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

Electric conductors:
The electric field inside a conductor is zero. All excess charge on a conductor resides on the surface of that conductor.

Electric Potential:
The electric potential is defined as the potential energy per unit charge. If the electric potential at some point is $V$ then the electric potential energy at that point is $U = qV$. The electric potential function $V(\vec{r})$ is given by the line integral:
$$V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{l} + V(\vec{r}_0)$$
Beware of the minus sign. This gives the potential produced by a point charge $q$:
$$V = \frac{q}{4\pi\epsilon_0 r}$$
for a collection of charges $q_i$
$$V = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i}$$
and for a continuous distribution of charge
$$V = \int \frac{dq}{4\pi\epsilon_0 r}$$
where in each of these cases, the potential is taken to be zero infinitely far from the charges.

Field from potential:
If the electric potential function is known, the vector electric field can be derived from it:
$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$
or in vector form:
$$\vec{E} = -\left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)$$
Beware of the minus sign.