These are your formula sheets

DO NOT TURN IT IN!

Derivatives:

\[
\frac{d}{dx} ax^n = an x^{n-1}
\]

\[
\frac{d}{dx} \sin ax = a \cos ax
\]

\[
\frac{d}{dx} \cos ax = -a \sin ax
\]

\[
\frac{d}{dx} e^{ax} = ae^{ax}
\]

\[
\frac{d}{dx} \ln ax = \frac{1}{x}
\]

Integrals:

\[
\int ax^n \, dx = a \frac{x^{n+1}}{n+1}
\]

\[
\int \frac{dx}{x} = \ln x
\]

\[
\int \sin ax \, dx = -\frac{1}{a} \cos ax
\]

\[
\int \cos ax \, dx = \frac{1}{a} \sin ax
\]

\[
\int e^{ax} \, dx = \frac{1}{a} e^{ax}
\]

\[
\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a}
\]

\[
\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left( \sqrt{x^2 + a^2} + x \right)
\]

\[
\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a}
\]

\[
\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2 \sqrt{x^2 + a^2}}
\]

\[
\int \frac{x \, dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}
\]

DO NOT TURN THESE SHEETS IN!
Forces:
The force on a charge \( q \) moving with velocity \( \vec{v} \) in a magnetic field \( \vec{B} \) is
\[
\vec{F} = q\vec{v} \times \vec{B}
\]
and the force on a differential segment \( d\vec{l} \) carrying current \( I \) is
\[
d\vec{F} = I d\vec{l} \times \vec{B}
\]
Magnetic Flux:
Magnetic flux is defined analogously to electric flux (see formula sheet 1)
\[
\Phi_B = \int \vec{B} \cdot d\vec{A}
\]
The magnetic flux through a closed surface seems to be zero
\[
\oint \vec{B} \cdot d\vec{A} = 0
\]
Magnetic dipoles:
A current loop creates a magnetic dipole \( \vec{\mu} = I \vec{A} \) where \( I \) is the current in the loop and \( \vec{A} \) is a vector normal to the plane of the loop and equal to the area of the loop. The torque on a magnetic dipole in a magnetic field is
\[
\vec{\tau} = \vec{\mu} \times \vec{B}
\]
Biot-Savart Law:
The magnetic field \( d\vec{B} \) produced at point \( P \) by a differential segment \( d\vec{l} \) carrying current \( I \) is
\[
d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}
\]
where \( \hat{r} \) points from the segment \( d\vec{l} \) to the point \( P \).

Magnetic field produced by a moving charge:
Similarly, the magnetic field produced at a point \( P \) by a moving charge is
\[
\vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2}
\]
Ampere’s Law: (without displacement current)
\[
\oint \vec{B} \cdot d\vec{I} = \mu_0 I_{\text{encl}}
\]
Faraday’s Law:
The EMF produced in a closed loop depends on the change of the magnetic flux through the loop
\[
\mathcal{E} = -\frac{d\Phi_B}{dt}
\]
When an EMF is produced by a changing magnetic flux there is an induced, nonconservative, electric field \( \vec{E} \) such that
\[
\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_A \vec{B} \cdot d\vec{A}
\]
Mutual Inductance:
When a changing current \( i_1 \) in circuit 1 causes a changing magnetic flux in circuit 2, and vice-versa, the induced EMF in the circuits is
\[
\mathcal{E}_2 = -M \frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M \frac{di_2}{dt}
\]
where \( M \) is the mutual inductance of the two loops
\[
M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_1}
\]
where \( N_i \) is the number of loops in circuit \( i \).

Self Inductance:
A changing current \( i \) in any circuit generates a changing magnetic field that induces an EMF in the circuit:
\[
\mathcal{E} = -L \frac{di}{dt}
\]
where \( L \) is the self inductance of the circuit
\[
L = N \frac{\Phi_B}{i}
\]
For example, for a solenoid of \( N \) turns, length \( l \), area \( A \), Ampere’s law gives \( B = \mu_0 (N/l)i \), so the flux is \( \Phi_B = \mu_0 (N/l)iA \), and so
\[
L = \mu_0 N^2 \frac{i}{l} A
\]

LR Circuits:
When an inductor \( L \) and a resistance \( R \) appear in a simple circuit, exponential energizing and de-energizing time dependences are found that are analogous to those found for \( RC \)-circuits. The time constant \( \tau \) for energizing an LR circuit is
\[
\tau = \frac{L}{R}
\]

LC Circuits:
When an inductor \( L \) and a capacitor \( C \) appear in a simple circuit, sinusoidal current oscillation is found with frequency \( f \) such that
\[
2\pi f = \frac{1}{\sqrt{LC}}
\]
Capacitance:
A capacitor is any pair of conductors separated by an insulating material. When the conductors have equal and opposite charges $Q$ and the potential difference between the two conductors is $V_{ab}$, then the definition of the capacitance of the two conductors is

$$C = \frac{Q}{V_{ab}}$$

The energy stored in the electric field is

$$U = \frac{1}{2}CV^2$$

If the capacitor is made from parallel plates of area $A$ separated by a distance $d$, where the size of the plates is much greater than $d$, then the capacitance is given by

$$C = \frac{\varepsilon_0 A}{d}$$

Capacitors in series:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$$

Capacitors in parallel:

$$C_{\text{eq}} = C_1 + C_2 + ...$$

If a dielectric material is inserted, then the capacitance increases by a factor of $K$ where $K$ is the dielectric constant of the material

$$C = KC_0$$

Current:
When current flows in a conductor, we define the current as the rate at which charge passes:

$$I = \frac{dQ}{dt}$$

We define the current density as the current per unit area, and can relate it to the drift velocity of charge carriers by

$$\vec{J} = nq\vec{v}_d$$

where $n$ is the number density of charges and $q$ is the charge of one charge carrier.

Ohm’s Law and Resistance:
Ohm’s Law states that a current density $J$ in a material is proportional to the electric field $E$. The ratio $\rho = E/J$ is called the resistivity of the material. For a conductor with cylindrical cross section, with area $A$ and length $L$, the resistance $R$ of the conductor is

$$R = \frac{\rho L}{A}$$

A current $I$ flowing through the resistor $R$ produces a potential difference $V$ given by

$$V = IR$$

Resistors in series:

$$R_{\text{eq}} = R_1 + R_2 + ...$$

Resistors in parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$$

Power:
The power transferred to a component in a circuit by a current $I$ is

$$P = VI$$

where $V$ is the potential difference across the component.

Kirchhoff’s rules:
The algebraic sum of the currents into any junction must be zero:

$$\sum I = 0$$

The algebraic sum of the potential differences around any loop must be zero.

$$\sum V = 0$$

RC Circuits:
When a capacitor $C$ is charged by a battery with EMF given by $\mathcal{E}$ in series with a resistor $R$, the charge on the capacitor is

$$q(t) = CE \left( 1 - e^{-t/RC} \right)$$

where $t = 0$ is when the charging starts.

When a capacitor $C$ that is initially charged with charge $Q_0$ discharges through a resistor $R$, the charge on the capacitor is

$$q(t) = Q_0 e^{-t/RC}$$

where $t = 0$ is when the discharging starts.
Physics 208 — Formula Sheet for Exam 1
Do NOT turn in these formula sheets!

Force on a charge:
An electric field $\vec{E}$ exerts a force $\vec{F}$ on a charge $q$ given by:

$$\vec{F} = q\vec{E}$$

Coulomb’s law:
A point charge $q$ located at the coordinate origin gives rise to an electric field $\vec{E}$ given by

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2} \hat{r}$$

where $r$ is the distance from the origin (spherical coordinate), $\hat{r}$ is the spherical unit vector, and $\varepsilon_0$ is the permittivity of free space:

$$\varepsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$$

Superposition:
The principle of superposition of electric fields states that the electric field $\vec{E}$ of any combination of charges is the vector sum of the fields caused by the individual charges

$$\vec{E} = \sum_i \vec{E}_i$$

To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements and integrate all these elements:

$$\vec{E} = \int d\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2} \hat{r}$$

Electric flux:
Electric flux is a measure of the “flow” of electric field through a surface. It is equal to the product of the area element and the perpendicular component of $\vec{E}$ integrated over a surface:

$$\Phi_E = \int E \cos \phi \, dA = \int \vec{E} \cdot \hat{n} \, dA = \int \vec{E} \cdot d\vec{A}$$

where $\phi$ is the angle from the electric field $\vec{E}$ to the surface normal $\hat{n}$.

Gauss’ Law:
Gauss’ law states that the total electric flux through any closed surface is determined by the charge enclosed by that surface:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$$

Electric conductors:
The electric field inside a conductor is zero. All excess charge on a conductor resides on the surface of that conductor.

Electric Potential:
The electric potential is defined as the potential energy per unit charge. If the electric potential at some point is $V$ then the electric potential energy at that point is $U = qV$. The electric potential function $V(\vec{r})$ is given by the line integral:

$$V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{r} + V(\vec{r}_0)$$

Beware of the minus sign. This gives the potential produced by a point charge $q$:

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

for a collection of charges $q_i$

$$V = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i}$$

and for a continuous distribution of charge

$$V = \int \frac{dq}{4\pi\varepsilon_0 r}$$

where in each of these cases, the potential is taken to be zero infinitely far from the charges.

Field from potential:
If the electric potential function is known, the vector electric field can be derived from it:

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

or in vector form:

$$\vec{E} = -\left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)$$

Beware of the minus sign.