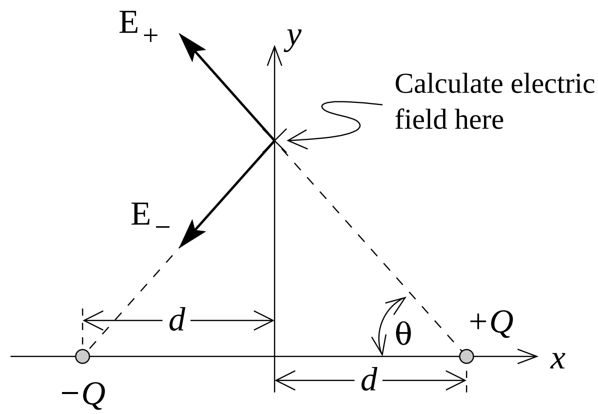
Problem 1 (Nearly identical to example 21.9. Very similar to homework problem 21-46. Similar to homework problem 21-23, but easier.)



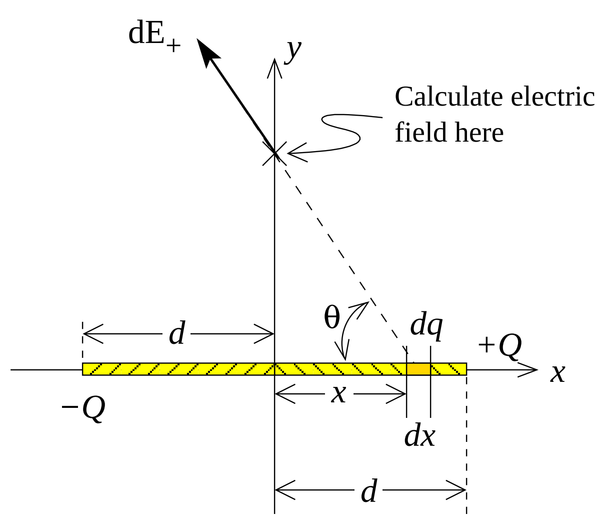
Looking at the Figure, we let E_+ be the field produced by $+Q$, and E_- be the field produced by $-Q$. These charges have the same magnitude, and are both the same distance from the point of interest, so the magnitudes of the fields produced by them are equal: $E_+ = E_-$.

$$E_+ = E_- = \frac{Q}{4\pi\epsilon_0 \left(\sqrt{y^2+d^2}\right)^2}$$

Furthermore, it is obvious that the y-components of the two fields cancel, and the x-components add. Thus,

$$\begin{aligned} E_y &= \underline{\underline{0}} \\ E_x &= 2E_{+x} = 2E_+\cos\theta = 2\,\frac{Q}{4\pi\epsilon_0\left(\sqrt{y^2+d^2}\right)^2}\,\frac{d}{\sqrt{y^2+d^2}} = \underline{\underline{\frac{2Qd}{4\pi\epsilon_0\,(y^2+d^2)^{3/2}}}} \end{aligned}$$

Problem 2 (This was very similar to example 21.22 and homework problem 21-87.)



This is very similar to problem 1, but instead of a charge $+Q$ located a distance d from the origin, we have to consider a portion dq that is located a distance x from the origin, and then add up (integrate) the contributions for all values of x from 0 to d. This will give us the field E_+ from the positive rod:

$$\begin{aligned} E_{+x} &= \int dE_+\ \cos\theta \\ &= \int \frac{dq}{4\pi\epsilon_0\left(\sqrt{y^2+x^2}\right)^2}\,\frac{x}{\sqrt{y^2+x^2}} \\ &= \int_0^d \frac{\frac{Q}{d}dx\,x}{4\pi\epsilon_0\,(y^2+x^2)^{3/2}} \\ &= \frac{Q}{4\pi\epsilon_0 d}\left[\frac{-1}{\sqrt{y^2+x^2}}\right]_0^d = \frac{Q}{4\pi\epsilon_0 d}\left[\frac{1}{y} - \frac{1}{\sqrt{y^2+d^2}}\right] \end{aligned}$$

Just as before $E_- = E_+$, and so $\underline{\underline{E_y = 0}}$ and $E_x = 2E_{+x} = \underline{\underline{\frac{Q}{2\pi\epsilon_0 d}\left[\frac{1}{y} - \frac{1}{\sqrt{y^2+d^2}}\right]}}$

Problem 3 (This was basically identical to homework problem 22-6.)

Gauss' Law tell us the flux is the enclosed charge divided by ϵ_0. We just look at the total charge enclosed by each of the surfaces:

(a) $\Phi = (q_1 + q_2 + q_3)/\epsilon_0$ (b) $\Phi = 0$ (c) $\Phi = (q_1 + q_3)/\epsilon_0$
(d) $\Phi = (q_1 + q_2)/\epsilon_0$ (e) $\Phi = (q_0)/\epsilon_0$

Problem 4 (Parts a and b were basically like homework 22-14. Part c was basically like homework 22-36. The whole thing was basically the same as homework 22-42.)

We can solve each part with Gauss' Law. We use a spherical gaussian surface of radius r concentric with the charged spheres:

$$\Phi = \frac{Q_{\text{encl}}}{\epsilon_0} \quad \Longrightarrow \quad E\,4\pi r^2 = \frac{Q_{\text{encl}}}{\epsilon_0} \quad \Longrightarrow \quad E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2}$$

(a) No charge in a conductor, so $Q_{\text{encl}} = 0$ so $\underline{\underline{E = 0}}$.

(b) $Q_{\text{encl}} = -Q$ so $\underline{\underline{E = \dfrac{-Q}{4\pi\epsilon_0 r^2}}}$ where the minus sign means the field is pointing in.

(c) This is the tough one. The enclosed charge is the entire $-Q$ sphere plus the part of the $+Q$ sphere that is in our volume. Let ρ be the charge density of the outer spherical shell:

$$\rho = \frac{Q}{\frac{4}{3}\pi R_3^3 - \frac{4}{3}\pi R_2^3}$$

then

$$Q_{\text{encl}} = -Q + \rho V_{\text{encl}} = -Q + \rho\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R_2^3\right) = Q\left(-1 + \frac{r^3 - R_2^3}{R_3^3 - R_2^3}\right) = -Q\,\frac{R_3^3 - r^3}{R_3^3 - R_2^3}$$

Thus

$$\underline{\underline{E = \frac{-Q}{4\pi\epsilon_0 r^2}\,\frac{R_3^3 - r^3}{R_3^3 - R_2^3}}}$$

(d) Now the enclosed charge is the $-Q$ inner sphere and the entire $+Q$ outer sphere, so $Q_{\text{encl}} = 0$ so $\underline{\underline{E = 0}}$.

Problem 5 (Very similar to homework 23-56.)

The field between two large parallel plates is constant, so $V = E\,d \;\Rightarrow\; E = V/d$.

The force on the charge from the electric field is $F = qE = qV/d$.

In equilibrium, this must equal the force from the spring: $qV/d = kx$. So $\underline{\underline{x = \dfrac{qV}{kd}}}$.

Problem 6 (Part a was just like example 23-11 and homework 23-68. Part b is just thinking. Part c was like homework problems 23-3 or 23-13.)

(a) We can just add up a bunch of differential elements along the bent rod. For each element $dV = dq/(4\pi\epsilon_0 R)$ so we have

$$\begin{aligned} V &= \int dV = \int \frac{dq}{4\pi\epsilon_0 R} = \frac{1}{4\pi\epsilon_0 R}\int dq \qquad \text{(because R is constant)} \\ &= \underline{\underline{\frac{Q}{4\pi\epsilon_0 R}}} \end{aligned}$$

(b) Look at the symmetry of the bent rod. The charge will move away from the center of the rod, so along a line $y = -x$.

(c) Use conservation of energy:

$$E_i = E_f \quad \Rightarrow \quad K_i + U_i = K_f + U_f \quad \Rightarrow \quad 0 + qV_i = \frac{1}{2}mv_f^2 + qV_f$$

$$0 + q\frac{Q}{4\pi\epsilon_0 R} = \frac{1}{2}mv_f^2 + 0 \quad \Rightarrow \quad \underline{\underline{v_f = \sqrt{\frac{2qQ}{4\pi\epsilon_0 mR}}}}$$

Problem 7 (This was just like homework 23-42, and similar to 23-46.)

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{1}{4\pi\epsilon_0}\frac{2q}{\sqrt{a^2+x^2}}\right) = -\frac{2q}{4\pi\epsilon_0}\frac{\partial}{\partial x}\left(a^2+x^2\right)^{-1/2} \\ &= -\frac{2q}{4\pi\epsilon_0}\left(-\frac{1}{2}\right)\left(x^2+a^2\right)^{-3/2}(2x) \\ &= \underline{\underline{\frac{2qx}{4\pi\epsilon_0\left(x^2+a^2\right)^{3/2}}}} \\ E_y &= -\frac{\partial V}{\partial y} = \underline{\underline{0}} \\ E_z &= -\frac{\partial V}{\partial z} = \underline{\underline{0}} \end{aligned}$$