Problem 1.  (See chapter 5, problem 4; and chapter 6, problem 3 and exercise 4.)

(a) There are 4 surfaces. Charge $Q$ is placed on the inner sphere, so it moves to the outer surface of the inner sphere. So, the inside surface at $r = A$ has charge $q = 0$ and the outside surface at $r = A + T$ has charge $+Q$.

Now, there is no electric field in the outside conductor, so the inside surface at $r = B$ must have charge $-Q$ and the outside surface at $r = B + T$ therefore has charge $+Q$.

Thus, the charge per unit areas are:

$$\sigma = \begin{cases} 
0 & r = A \\
\frac{Q}{4\pi (A+T)^2} & r = A + T \\
\frac{-Q}{4\pi (B)^2} & r = B \\
\frac{Q}{4\pi (B+T)^2} & r = B + T 
\end{cases}$$

(b) $$\Delta V = V(A) - V(B + T) = \int_{A}^{B+T} \vec{E} \cdot d\vec{r} = \int_{A}^{A+T} 0 \, dr + \int_{A+T}^{B} \frac{Q}{4\pi \epsilon_0 r} \, dr + \int_{B}^{B+T} 0 \, dr$$

$$= \frac{Q}{4\pi \epsilon_0} \left[ -\frac{1}{r} \right]_{A+T}^{B} = \frac{Q}{4\pi \epsilon_0} \left[ \frac{1}{A + T} - \frac{1}{B} \right]$$

(c) $$C = \frac{Q}{V} = \frac{4\pi \epsilon_0}{\frac{1}{A+T} - \frac{1}{B}}$$

Problem 2.  (See Chapter 5, problem 1.)

(a) $$\Phi = \frac{Q_{\text{enc}}}{\epsilon_0} \Rightarrow EA = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$r < A:$  
$$E 2\pi rL = \frac{Q}{\epsilon_0} \frac{\pi r^2 L}{\pi A^2 L} \Rightarrow E = \frac{Q}{2\pi \epsilon_0 L} \frac{r}{A^2}$$

$r > A:$  
$$E 2\pi rL = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{2\pi \epsilon_0 r L}$$

(b) $$V(0) - V(5A) = \int_{0}^{5A} \vec{E} \cdot d\vec{r} = \int_{0}^{A} \frac{Q}{2\pi \epsilon_0 L} \frac{r}{A^2} \, dr + \int_{A}^{5A} \frac{Q}{2\pi \epsilon_0 L} \frac{dr}{r}$$

$$= \frac{Q}{2\pi \epsilon_0 L} \left( \frac{1}{2} \ln 5 \right)$$
Problem 3. (See chapter 8, exercise 2, and problem 6.)

I’ll call the current in the top wire $i_1$ positive to the right, in the middle wire $i_2$ positive to the right, and in the lower wire $i_3$ positive to the left.

The continuity equation gives:

$$i_1 + i_2 = i_3$$

For the loop equations, I’ll go clockwise starting on the left. We get, for the top loop, bottom loop, and outer loop:

$$-i_1 R + V - V + i_2 R = 0$$
$$-i_2 R + V - i_3 R_3 = 0$$
$$-i_1 R + V - i_3 R_3 = 0$$

To solve these, we need the first one, and any two of the last three. Doing the very simple algebra, we find:

$$i_3 = \frac{V}{R_3 + R/2}$$

(b) If I replace the top resistor by a capacitor, then $i_1 = 0$ and $i_2 = i_3$. So, I’ll just call $i_2$ and $i_3$ simply $i$. The lower loop equation is:

$$-i R + V - i R_3 = 0 \Rightarrow i = \frac{V}{R + R_3}$$

Looking at the top wire and the middle wire, we see that the potential change across the capacitor is the same as across $R$, so $Q = CV$ where $V = i R$, or

$$Q = C \frac{V}{R + R_3} R$$

Problem 4. (See chapter 7 problem 4 and exercise 4.)

(a) For the first 2/3 and the last 2/3, $j = i/A$. For the middle 1/3, $j = i/(2A)$.

(c) We can use $R = \rho l / A$ but we have to break it into pieces of constant cross-sectional area:

$$R = \rho \frac{2W}{A} + \rho \frac{1W}{2A} + \rho \frac{2W}{A} = \frac{\rho W}{A} \left( \frac{2}{3} + \frac{1}{6} + \frac{2}{3} \right) = \frac{3\rho W}{2A}$$

(b) We can use $V = i R$ and just use $R$ from above:

$$V = 3 \frac{i \rho W}{2A}$$