Problem 1. Force on a current in a Magnetic Field: Section 27-6 and Problem 27.34. Also see Quiz 10.

The force on a current-carrying wire is

\[ \vec{F} = I \vec{\ell} \times \vec{B} \, . \]

So, the force on the vertical segment of length \( a \) points in the \( x \) (horizontal) direction and has magnitude \( F_x = IaB \). Similarly, the force on the horizontal segment is in the vertical direction and has magnitude \( F_y = IbB \).

The current has magnitude \( I = V/R = 50 \text{ A} \) so the components are \( F_x = 50 \times .03 \times 2 = 3 \text{ N} \) and \( F_y = 4 \text{ N} \). Thus, the magnitude of the net force is

\[ F = \sqrt{F_x^2 + F_y^2} = \sqrt{3^2 + 4^2} = 5 \text{ N} \, . \]

Problem 2. (Identical to assigned homework 28.69.)

The field produced by any small segment of the wire is given by

\[ d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{ds \times \hat{r}}{r^2} \]

where \( d\vec{l} \) points along the segment of the wire, and \( \vec{r} \) points from \( d\vec{l} \) to the place you want the field. For the two long pieces of wire, this cross product is zero, so the field is only produced from the circular arc. To calculate this, we use the following figure where \( d\vec{l} \times \hat{r} = dl = Sd\theta \) into the page:

\[ B = \frac{\mu_0 i}{4\pi} \frac{S \pi/2}{2S} = \frac{\mu_0 i \pi}{8S} \text{ (into the page)} \]

Problem 4 (Simplification of assigned Problem 30.69.)

(a) Just before we close the switch, no current is flowing. The inductor resists changes in current, so just after the switch is closed, the current will still be zero.

(b) After a long time, the inductor is fully energized, and so acts as a short. The potential \( V \) just drops across the two resistors so the current is \( I = V/(R_1 + R_2) \).

(c) When we close this switch, the energized inductor will start to decay. Initially, the current is whatever it was when the switch is closed, so the same as part (b).

(d) The switch \( S_2 \) is shorting the inductor across \( R_2 \), so the inductor decays across \( R_2 \) only.

\[ i(t) = \frac{V}{R_1 + R_2} e^{-t/\tau} \quad \text{where} \quad \tau = L/R_2 \]
Problem 3. (Variation of Problem 29.45.)

We’re going to use Faraday’s law, so first let’s calculate the flux through the loop.

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \, dA = \int_H^{H+D} B(x) W \, dx \]

where \( B(x) \) is the field a distance \( x \) from the long straight wire. You can use Ampère’s law to get (or just remember) \( B(x) = \mu_0 i / 2\pi x \), so

\[ \Phi_B = \int_H^{H+D} \frac{\mu_0 i}{2\pi x} W \, dx = \frac{\mu_0 i W}{2\pi} \ln \frac{H + D}{H} \]

into the page.

Now, let’s look at Faraday’s law: \( E = -\frac{d}{dt} \Phi_B \). Since we calculated the flux into the page, this is the clockwise EMF induced in the loop. Let \( I \) be the induced current, we apply Kirchoff’s law going clockwise around the loop:

\[ E - IR = 0 \quad \Rightarrow \quad I(t) = -\frac{1}{R} \frac{d}{dt} \Phi_B \]

So

\[
I(t) = -\frac{1}{R} \frac{d}{dt} \frac{\mu_0 i W}{2\pi} \ln \frac{H + D}{H} = -\frac{\mu_0 W}{2\pi R} \ln \frac{H + D}{H} \frac{di}{dt} = -\frac{\mu_0 W}{2\pi R} \ln \frac{H + D}{H} \frac{d}{dt} i_0 \cos \omega t \\
= +\frac{\mu_0 i_0 \omega W}{2\pi R} \ln \frac{H + D}{H} \sin \omega t \quad \text{(positive clockwise)}
\]

Problem 5. (See Example 30.9.)

\[ \omega = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad LC = \frac{1}{\omega^2} = \frac{1}{(2\pi f)^2} = \frac{1}{(2\pi 10^6)^2} = \frac{10^{-12}}{4\pi^2} \approx \frac{10^{-13}}{4} = 2.5 \times 10^{-14} = 25 \times 10^{-15} \]

But \( 25 \times 10^{-15} = 25 \times 10^{-6} \cdot 1 \times 10^{-9} \) so pick \( L = 25 \times 10^{-6} \) H and \( C = 10^{-9} \) F.