Problem 1

(a) Here are the free-body diagrams with the $+x$-direction shown as given in the problem:

(b) Next step: Newton’s Laws. For mass $M_2$ and $M_1$ respectively we have:

\[ \Sigma F_x = P \cos \theta - T - f = M_2 a \]
\[ \Sigma F_x = T - M_1 g = M_1 a \]

Adding these equations and solving for $a$ gives

\[ a = \frac{P \cos \theta - f - M_1 g}{M_1 + M_2} \tag{1} \]

Now, we must substitute for $f$. We know $f = \mu N$ where $N$ is the normal force between mass $M_2$ and the plane. We can get the normal force by looking at Newton’s Law for this mass in the $y$-direction:

\[ \Sigma F_y = N - M_2 g - P \sin \theta = 0 \]

So $N = M_2 g + P \sin \theta$ which gives $f = \mu (M_2 g + P \sin \theta)$ which we substitute into Eq. (1) to get:

\[ a = \frac{P \cos \theta - \mu (M_2 g + P \sin \theta) - M_1 g}{M_1 + M_2} = \frac{P (\cos \theta - \mu \sin \theta) - (M_1 + \mu M_2) g}{M_1 + M_2} \]

Problem 2

There are two forces on the Enterprise — the force of attraction from mass $M_1$ and the force of attraction from mass $M_2$. These forces are shown in the figure:

So, the total force on the Enterprise is

\[ \vec{F}_{\text{total}} = \vec{F}_1 + \vec{F}_2 = G \frac{M_1 m}{b^2} \left( \cos \theta \hat{i} + \sin \theta \hat{j} \right) \]

where $m$ is the mass of the Enterprise. Since $\theta = 45^\circ$ we have $\cos \theta = \sin \theta = 1/\sqrt{2}$. Finally, we write the acceleration as

\[ \vec{a} = \frac{\vec{F}}{m} = \frac{G}{b^2} \left\{ \left( M_1 + \frac{M_2}{2\sqrt{2}} \right) \hat{i} + \frac{M_2}{2\sqrt{2}} \hat{j} \right\} \]
Problem 3

(a) \[ W = \int_{x_i}^{x_f} F(x) \, dx = \int_{-x_0}^{x_0} \left( Ax + Bx^2 - Cx \right) \, dx \]
\[ = A \left( \frac{(x_0)^2}{2} - \frac{(-x_0)^2}{2} \right) + B \left( \frac{(x_0)^3}{3} - \frac{(-x_0)^3}{3} \right) - C \left( \frac{(x_0)^4}{4} - \frac{(-x_0)^4}{4} \right) \]
\[ = 0 + 2B \frac{x_0^3}{3} + 0 = 2B \frac{x_0^3}{3} \]

(b) \[ W = K_f - K_i \]
\[ 2B \frac{x_0^3}{3} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \]
\[ 2B \frac{x_0^3}{3} = \frac{1}{2}mv_f^2 - 0 \quad \Rightarrow \quad v_f = \sqrt{\frac{4Bx_0^3}{3m}} \]

Problem 4

The only force in the \( x \)-direction is friction. The friction force points in the \(-x\)-direction, so \( f = -\mu N = -\mu Mg \). But \( \mu = \mu(x) \) is a function of position, so we have a single, position dependent force in the \( x \) direction given by
\[ f(x) = -\mu_0 Mg \left( 1 - \frac{x}{L} \right) \]

We know the starting position, \( x_i = 0 \) and the final position \( x_f = L \) and we know the starting velocity \( v_i = v_0 \) and we want the final velocity \( v_f \). So, this problem is exactly like the preceding one!

\[ W = \int_{x_i}^{x_f} F(x) \, dx = \int_{0}^{L} -\mu_0 Mg \left( 1 - \frac{x}{L} \right) \, dx \]
\[ = -\mu_0 Mg \left\{ \left( \int_{0}^{L} 1 \, dx \right) - \left( \int_{0}^{L} \frac{x}{L} \, dx \right) \right\} \]
\[ = -\mu_0 Mg (L - 0) + \mu_0 Mg \left( \frac{L^2}{2L} - 0 \right) \]
\[ = -\mu_0 MgL + \mu_0 Mg \frac{L}{2} \]
\[ = -\mu_0 Mg \frac{L}{2} \]

So \( -\mu_0 Mg \frac{L}{2} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \quad \Rightarrow \quad v_f = \sqrt{v_0^2 - \mu_0gL} \).