Problem 1

(a) \( a = \frac{dv}{dt} \) so we want the slope of the line at \( t = 5 \) s. The slope is the rise over the run: \( a = \frac{(10 \text{ m/s} - 0 \text{ m/s})}{(10 \text{ s} - 0 \text{ s})} = 1 \text{ m/s}^2 \). At \( t = 15 \) s, the line is flat, the slope is zero, so \( a = 0 \).

(b) \( x = x_0 + \int_0^t v(t) \, dt \). We can just take \( x_0 = 0 \) since we only want the total distance covered. So we just need the integral, or the area under the curve. This is a triangle plus a rectangle, or \( x = (10 \text{ m/s} \times 10 \text{ s})/2 + (10 \text{ m/s} \times 5 \text{ s}) = 100 \text{ m} \).

Problem 2

\[
x = x_0 + At^{3/2} - \frac{1}{3}Ct^3
\]
\[
v = \frac{dx}{dt} = \frac{3}{2}At^{1/2} - Ct^2
\]
\[
a = \frac{dv}{dt} = \frac{3}{4}At^{-1/2} - 2Ct
\]

Problem 3

\[
v(t) = v_0 + \int_0^t a(t) \, dt
\]
\[
= v_0 + \int_0^t (-At) \, dt
\]
\[
= v_0 - \frac{1}{2}At^2
\]
\[
x(t) = x_0 + \int_0^t v(t) \, dt
\]
\[
= 0 + \int_0^t \left( v_0 - \frac{1}{2}At^2 \right) \, dt
\]
\[
= v_0 t - \frac{1}{6}At^3
\]
Problem 4
(a) We want $x$ when $v = 0$, so let’s find out when $v = 0$:
\[
v = 0 \quad \Rightarrow \quad v_0 = \frac{1}{2}At^2 \quad \Rightarrow \quad t = \sqrt{\frac{2v_0}{A}}.
\]
Now, let’s see what $x$ is at this time:
\[
x = v_0 t - \frac{1}{6}At^3 = v_0 \sqrt{\frac{2v_0}{A}} - \frac{1}{6}A \left( \sqrt{\frac{2v_0}{A}} \right)^3
\]
\[
= v_0 \sqrt{\frac{2v_0}{A}} - \frac{1}{6}A \sqrt{2v_0} \frac{2v_0}{A}
\]
\[
= v_0 \sqrt{\frac{2v_0}{A}} - \frac{1}{3}v_0 \sqrt{\frac{2v_0}{A}} = 2 \frac{v_0 \sqrt{2v_0}}{A}
\]
(b) Want $v$ when $x = 0$, so let’s find out when $x = 0$:
\[
x = 0 \quad \Rightarrow \quad v_0 t = \frac{1}{6}At^3 \quad \Rightarrow \quad t = \sqrt{\frac{6v_0}{A}}.
\]
Now, let’s see what $v$ is at this time:
\[
v = v_0 - \frac{1}{2}At^2 = v_0 - \frac{1}{2}A \frac{6v_0}{A}
\]
\[
= v_0 - 3v_0 = -2v_0.
\]

Problem 5
First note that $v_{0x} = v_0 \cos \theta$ and $v_{0y} = v_0 \sin \theta$. Now:
\[
x = x_0 + v_{0x} t \quad \Rightarrow \quad R = 0 + v_0 \cos \theta t\]
and
\[
y = y_0 + v_{0y} t - \frac{1}{2}gt^2 \quad \Rightarrow \quad 0 = H + v_0 \sin \theta t - \frac{1}{2}gt^2.
\]
Solve the first for $t$ and substitute into the second:
\[
t = \frac{R}{v_0 \cos \theta} \quad \Rightarrow \quad 0 = H + v_0 \sin \theta \frac{R}{v_0 \cos \theta} - \frac{1}{2}g \left( \frac{R}{v_0 \cos \theta} \right)^2
\]
So
\[
H = \frac{1}{2}g \left( \frac{R}{v_0 \cos \theta} \right)^2 - R \tan \theta.
\]