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**IMPORTANT**

Read these directions carefully:

- There are 8 problems totalling 200 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.

- **Indicate what you are doing!** We cannot give full credit for merely writing down the answer. **Neatness counts!** I will give generous partial credit if I can tell that you are on the right track. This means you must be neat and organized.

- Each problem with its associated figure is self explanatory. If you must ask a question, then come to the front, being as discrete as possible so as not to disturb others.

- Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.
Problem 1. 25 points.

The football team at the Texas Atomic and Molecular University was recently embarrassed when an opponent ran a very clever route during a play. Unfortunately, no video records of the play were kept. However, a technician at the field was testing radar guns for the baseball team at the time, and he recorded both the $x$- and $y$-components of the opposing player’s velocity.

As a consulting engineer, you job is to analyze the play for the coach, whose job is on the line. You model the play in the following way:

- Choose $t = 0$ to correspond to the start of the play.
- Choose the $x$-axis to be along the line of scrimmage, with the origin at the player in question.
- Choose the $y$-axis to be in the forward direction along the field, with the origin at the player in question.

These choices are shown schematically in the following figure:

In this case, the velocity of the opposing player is well modeled by the following equation:

$$\vec{v}(t) = At\hat{i} + (B - Ct^2)\hat{j}$$

where $A$, $B$, and $C$ are positive constants.

(a) (7 points) Derive an equation for the acceleration vector of the player as a function of time.
(b) (8 points) Derive an equation for the position vector of the player as a function of time.
(c) (5 points) Calculate how far forward the player gets before coming back toward the $x$-axis. (That is, calculate the maximum value of $y$ obtained by the player.)
(d) (5 points) Calculate the speed of the player when he returns to the $x$-axis.

You may use the next page to complete this problem.
Print your name:  

Physics 218:  Mechanics, Final Exam

Problem 1,  *Continued*...
Problem 2. 25 points.

Two boxes connected by a light rope slide with friction along a flat surface tilted at an angle $\theta$ from the horizontal. The boxes are made of different materials and have different coefficients of kinetic friction with the surface. The upper box has mass $M$ and coefficient of kinetic friction $\mu$. The lower box has mass $2M$ (twice the mass of the upper box) and coefficient of kinetic friction $\mu/4$ (one-fourth as large as for the upper box). Assume that $\mu$ is low enough that the boxes slip down the plane with the rope taut. Calculate the acceleration of the boxes.
Problem 3. 25 points.

A spring of force constant $k$ is connected with one end to a fixed wall and the other end to a light rope. The rope is connected via a massless pulley to a block of mass $M$.

(a) If the mass is released, calculate the maximum distance that the spring is stretched.
(b) At the moment that the spring is stretched this maximum distance, calculate the acceleration of the block.

Note: The rope does not stretch.
Problem 4. 25 points.

A truck of mass $M$ has lost its brakes, and taken the emergency ramp off a highway. This ramp leads to a trench filled with sand that stops the truck. The effect of the sand stopping the truck can be modeled as a particle sliding on a horizontal surface with friction. Initially, the coefficient of kinetic friction between the truck and the sand is $\mu_0$, but as the truck moves down the trench, the tires sink into the sand, and the coefficient of friction increases as the square of the distance traveled, such that after the truck has gone a distance $L$ the coefficient of friction will have increased by an amount $\mu_1$.

This is shown in the following picture:

\[ \mu(x) = \mu_0 + \mu_1 \left( \frac{x}{L} \right)^2 \]

Suppose the truck goes a distance $D$ down the trench before coming to rest. Calculate the initial velocity $v_0$ of the truck as it entered the trench.
Problem 5. 25 points.

A particle moves in one dimension under the influence of only conservative forces. The total force on the particle is well modeled by an anharmonic force law:

\[ F(x) = -kx + bx^2 \]

where \( k \) and \( b \) are given constants.

(a) Calculate a potential energy function for the particle. Take the potential energy to be zero at a point where the force is zero. The potential energy function you should get may look something like the graph below.

(b) Describe the two labeled positions: \( x = 0 \) and \( x = k/b \).

(c) Suppose that the particle is at \( x_0 = 0 \) with initial speed \( v_0 = \sqrt{k^3/(mb^2)} \). Calculate what speed the particle will have when it reaches the point \( x = k/b \).

Remember: You must show your work. Do not just write down the answer. Work Neatly!
Problem 6. 25 points.

A rifle bullet of mass $m$ strikes and embeds itself in a block of mass $M$ that rests on a frictionless horizontal surface and is attached to a spring of force constant $k$. The impact compresses the spring a maximum distance $S$. Calculate the velocity of the bullet just before it impacted the block.

If I can read it, I may find more points! Be neat!
A steel ball-bearing is allowed to roll without slipping inside a hemispherical bowl. The radius of the ball-bearing is exactly equal to $1/5$ the radius of the bowl. Suppose the ball-bearing is placed inside the bowl, with its center just even with the lip of the bowl, and released from rest. It then rolls without slipping down the bowl, through the bottom and back up the other side.

Calculate the normal force of contact between the ball-bearing and the bowl as the ball-bearing passes through the bottom of the bowl.

Treat the ball-bearing as a uniform solid sphere with moment of inertia $I = \frac{2}{5}MR^2$.

Hint: Try to work neatly.
Problem 8. 25 points.

A large flywheel of mass \( M \) and radius \( R \) is mounted so it can spin freely about its axis. A light strong rope is wrapped around the flywheel, and a block of the same mass \( M \) is attached to the rope. When the block is released, the flywheel starts to spin. Treat the flywheel as a uniform solid cylinder with moment of inertia \( I = MR^2/2 \).

Calculate the magnitude of the acceleration of any point on the outside edge of the flywheel after the block has fallen through a distance equal to the radius of the wheel. That is, after the block has dropped a distance \( R \).

*Hint:* Acceleration is a vector. In this case it has two components: the radial component depends on the angular velocity of the wheel, and the tangential component can be related to the acceleration of the block.

*Hint 2:* Both the block and the flywheel have the same mass \( M \). Do not assign different masses to the two.

Did you work neatly? Neatness is important.
Potentially useful equations

Calculus:

Derivatives: If \( x(t) = C t^n \) then
\[
\frac{dx}{dt} = C n t^{n-1}
\]

Integrals:

\[
\int_{t_1}^{t_2} C t^n \, dt = C \left[ \frac{t_2^{n+1}}{n+1} - \frac{t_1^{n+1}}{n+1} \right]
\]