Problem 1
(a) We use the formula \( a = \frac{\Delta v}{\Delta t} \). It is easy to use the interval \( 20 < t < 40 \) since the slope is constant there, so

\[
a(25\,\text{s}) = \frac{\Delta v}{\Delta t} = \frac{0\,\text{m/s} - 20\,\text{m/s}}{40\,\text{s} - 20\,\text{s}} = -1\,\text{m/s}^2
\]

and this is the acceleration at any time in the interval \( 20\,\text{s} < t < 40\,\text{s} \).

(b) \[
x = x_0 + \int_0^{40} v(t)\,dt = 0 + \text{Area under curve} = 400 \text{ (rectangle)} + 200 \text{ (triangle)} = 600\,\text{m}
\]

Problem 2
(a) The term \( Bt^3 \) appears additively with \( x \) which has units of meters. Thus, \( Bt^3 \) must have units of meters, so \( B \) must have units of \( \text{m/s}^3 \). Similarly \( C \) must have units of \( \text{m/s}^4 \).

(b) We find:
\[
v(t) = \frac{dx}{dt} = 3Bt^2 - \frac{1}{4} C 4t^3 = 3Bt^2 - Ct^3.
\]
\[
a(t) = \frac{dv}{dt} = 6Bt - 3Ct^2.
\]

Problem 3
We want to know the velocity when the acceleration is zero. So, we set \( a(t) = 0 \) and solve for the time. We get \( a(t) \) from problem 2 part b:
\[
a(t) = 6Bt - 3Ct^2 = 0 \implies t = \frac{2B}{C}.
\]

Now, we take that value for \( t \) and substitute it into the expression for \( v(t) \) given in problem 2 part b:
\[
v(t) = 3Bt^2 - Ct^3 = 3B \left( \frac{2B}{C} \right)^2 - C \left( \frac{2B}{C} \right)^3 = \frac{12B^3}{C^2} - \frac{8B^3}{C^2} = \frac{4B^3}{C^2}.
\]
Problem 4

\[ v(t) = v_0 + \int_0^t a(t) \, dt \]
\[ = v_0 + \int_0^t (2A + 20B \, t^3) \, dt \]
\[ = v_0 + 2At^2 + 5Bt^4 \]

\[ x(t) = x_0 + \int_0^t v(t) \, dt \]
\[ = x_0 + \int_0^t (v_0 + 2At + 5Bt^4) \, dt \]
\[ = x_0 + v_0t + At^2 + Bt^5 \]

Problem 5

(a) Resolve into rectangular components: We are given \( x_0 = 0, \ y_0 = 0, \ v_{x0} = 0, \) and \( v_{y0} = v_0. \) Furthermore,

\[ a_x(t) = At \quad a_y(t) = -g \]

Velocity:

\[ v_x = v_{x0} + \int_0^t a_x(t) \, dt \quad v_y = v_{y0} + \int_0^t a_y(t) \, dt \]
\[ = 0 + \int_0^t At \, dt \quad = v_0 + \int_0^t (-g) \, dt \]
\[ = \frac{1}{2}At^2 \quad = v_0 - gt \]

Position:

\[ x = x_0 + \int_0^t v_x(t) \, dt \quad y = y_0 + \int_0^t v_y(t) \, dt \]
\[ = 0 + \int_0^t \frac{1}{2}At^2 \, dt \quad = 0 + \int_0^t (v_0 - gt) \, dt \]
\[ = \frac{1}{6}At^3 \quad = v_0t - \frac{1}{2}gt^2. \]

So,

\[ \vec{r}(t) = \left( \frac{1}{6}At^3 \right) \hat{i} + \left( v_0t - \frac{1}{2}gt^2 \right) \hat{j} \]

(b) We want to know the value of \( x \) when \( y \) is equal to 0. So, we set the \( y \)-component to zero and solve for \( t \):

\[ v_0t - \frac{1}{2}gt^2 = 0 \quad \Rightarrow \quad t = \frac{2v_0}{g}. \]

And now, we calculate the range:

\[ x = \frac{1}{6}At^3 = \frac{1}{6}A \left( \frac{2v_0}{g} \right)^3 = \frac{2A}{3} \left( \frac{v_0}{g} \right)^3. \]