Print your full name: ________________________________

Sign your name: ________________________________

Please fill in your Student ID number: __ __ __ __ __ __ __

IMPORTANT

Read these directions carefully:

• There are 5 problems totalling 100 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.

• Indicate what you are doing! We cannot give full credit for merely writing down the answer. Neatness counts! I will give generous partial credit if I can tell that you are on the right track. This means you must be neat and organized.

• Each problem with its associated figure is self explanatory. If you must ask a question, then come to the front, being as discrete as possible so as not to disturb others.

• Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.
Problem 1. 15 points.

A particle of mass $m$ moves in 1 dimension under the influence of only conservative forces. The net force on the particle is given by:

$$F(x) = -kx + \alpha x^3$$

where $k$ and $\alpha$ are given positive constants.

Calculate a potential energy function for this particle assuming the potential energy to be zero at the point $x = 0$. 
Problem 2. 30 points.

A particle of mass $m$ moves in 1 dimension under the influence of only conservative forces. A potential energy function is:

$$U(x) = \frac{A}{x^2} - \frac{B}{x}$$

where $A$ and $B$ are given positive constants. This function is plotted here for your convenience:

It is useful to consider the units of the constants $A$ and $B$. Let $[\ ]$ denote “the units of” like we do in class. Then:

$$[A] = \text{energy} \times \text{length}^2$$
$$[B] = \text{energy} \times \text{length}$$

so

$$[A/B] = \text{length} ,$$

$$[B^2/A] = \text{energy} ,$$

and

$$[B^3/A^2] = \text{force} .$$

Suppose that the particle is at $x = 4A/B$, and is moving to the left with velocity

$$v = -\sqrt{\frac{7B^2}{8mA}} .$$

(a) (5 points) Describe the subsequent motion of the particle. Do not write a novel. Keep it short and clear. **Write Neatly.**

(b) (5 points) Calculate the total mechanical energy of the particle.
Problem 2. Continued...

(c) (10 points) Calculate the velocity that the particle will have when it reaches \( x = \frac{2A}{B} \). Compare this to the original velocity given above, and make sure that this makes sense with what you wrote in part (a).

(d) (10 points) Calculate the net force on the particle at \( x = \frac{A}{B} \). Get the sign right. Indicate whether the force is to the left or to the right (or zero).

Work Neatly!
Problem 3. 20 points.

A block of mass $M$ and velocity $v_0$ moves to the right along a frictionless horizontal surface. It undergoes an impulsive collision with a block of mass $2M$ which is at rest. Calculate the velocity of the larger block after the collision for the following two cases:

(a) (10 points) the collision is perfectly inelastic.

(b) (10 points) the collision is inelastic and the first block rebounds off the second with velocity $v_0/4$ to the left.
Problem 4. 15 points.

You are given expressions for the polar unit vectors in terms of the cartesian unit vectors:

\[ \hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j} \]
\[ \hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j} \]

Start with the position vector \( \vec{r} = r \hat{r} \) and calculate the acceleration vector \( \vec{a} \) expressed in polar coordinates.

You may need the derivatives:

\[ \frac{d}{d\theta} \cos \theta = -\sin \theta \quad \text{and} \quad \frac{d}{d\theta} \sin \theta = \cos \theta . \]

Show all the steps in the derivation — do not use any other results that have been shown in class.
Problem 5. 20 points.

A pendulum consists of a particle of mass $M$ connected to the end of a light strong rope of length $L$. The particle is moved so the rope is at an angle $\alpha$ from the vertical, and the particle is released from rest.

Calculate the tension in the rope at the moment that the particle reaches the lowest point of its path.
Potentially useful equations

Calculus:

Derivatives:

If \( x(t) = C t^n \) then \( \frac{dx}{dt} = C n t^{n-1} \)

Integrals:

\[
\int_{t_1}^{t_2} C t^n \, dt = C \left[ \frac{t_2^{n+1}}{n+1} - \frac{t_1^{n+1}}{n+1} \right]
\]