Print your full name: ________________________________

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IMPORTANT

Read these directions carefully:

• There are 8 problems totalling 200 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.

• Indicate what you are doing! We cannot give full credit for merely writing down the answer. Neatness counts! I will give generous partial credit if I can tell that you are on the right track. This means you must be neat and organized.

• Each problem with its associated figure is self explanatory. If you must ask a question, then come to the front, being as discrete as possible so as not to disturb others.

• Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.
Problem 1. 25 points.

Your engineering firm has just designed a new missile for the National Weather Service, but the first test flight did not go so well. The missile was released from rest from the end of a crane, a distance $H$ above the ground. During the initial part of the flight, before the fuel was expended, the acceleration vector as a function of time was found to be well modeled by

$$\vec{a}(t) = Bt \hat{i} + (At^2 - g) \hat{j}$$

where $A$ and $B$ are positive constants, $g$ is the acceleration due to gravity, and a Cartesian coordinate system with the $y$-axis vertical is used.

The path of the rocket is illustrated here:

Find the $x$-$y$ coordinates of the position of the rocket at the lowest point in its trajectory (see the Figure.)

If you work neatly I will find more partial credit for you!
Problem 2. 25 points.

A block of mass $M$ is pressed against a vertical wall by a constant applied force $P$. The coefficient of static friction between the wall and the block is $\mu$. What is the minimum value of $P$ such that the block does not slide down the wall?
A block of mass $M_2$ is placed on a smooth horizontal surface. Neglect friction between the block and this surface. A second block of mass $M_1$ is placed on top of the first block. The coefficient of kinetic friction between the two block is $\mu$. The two blocks are connected together by a light rope and a massless pulley as shown.

A constant horizontal force $P$ is applied to the bottom block. As the block moves to the left, due to the pulley the top block moves to the right.

Calculate the acceleration of the blocks.
Problem 4. 25 points.

You are designing a restraining system for jet fighters that land on naval aircraft carriers. This system consists of cables that attach to a hook on the tail of the fighter. As the cables are stretched, they apply a force on the aircraft that is opposite the direction of, and proportional to the fourth power of the distance that the cables are stretched. Your job is to find the proportionality constant which your boss needs for further analysis.

In a mockup that is created for testing, a jet fighter with mass $M$ and maximum thrust $P$ is connected to the restraining system. The aircraft is initially at rest and the test pilot applies full thrust, causing the plane to lurch forward, and travel a distance $D$ before being brought to rest and pulled back by the restraining cables.

You model the force on the aircraft by the following equation:

$$F(x) = P - kx^4$$

where $x$ is the distance the airplane has moved, $F(x)$ is the total force on the aircraft, $P$ is the thrust exerted by the engine, and $k$ is the proportionality constant of the restraining system that you are trying to find.

(a) (15 points) Derive an expression for $k$ in terms of the aircraft thrust $P$ and the measured displacement $D$.

(b) (10 points) Calculate the maximum speed the aircraft reaches during this short test. Express the answer in terms of $P$, $D$, and $M$.

Don’t forget to be neat!
A particle moves in one dimension under the influence of only conservative forces. The potential energy function for the particle may be written as:

\[ U(x) = \begin{cases} 
  U_0 \left( \frac{x}{a} \right)^2 & x \leq 0 \\
  2U_0 \left( \frac{x}{a} \right)^3 & x \geq 0
\end{cases} \]

where \( U_0 \) and \( a \) are given constants. This function is plotted here for your convenience.

Suppose the particle is released from rest at \( x = +a \).

(a) (8 points) Calculate the instantaneous acceleration of the particle the moment it is released.

(b) (9 points) Calculate the maximum velocity that the particle subsequently obtains.

(c) (8 points) Calculate the minimum (most negative) value of \( x \) that the particle ever reaches.

Remember: You must show your work. Do not just write down the answer. Work Neatly!
Problem 6. 25 points.

Two cars are traveling along perpendicular routes. The first car has mass $M_1$ and speed $v_1$ and the second car has mass $M_2$ and speed $v_2$. The cars collide at an intersection, and lock together and go sliding through an empty parking lot a distance $S$ until they come to rest.

What is the coefficient of kinetic friction between the cars and the parking lot?
Problem 7. 25 points.

A solid disk of mass $M$ and radius $R$ is fixed at the top of a ramp. The ramp makes an angle $\theta$ with the horizontal. A rope is wrapped around the disk, and then attached to the axle of another solid disk of mass $M$ and radius $R$. This second disk is allowed to roll without slipping down the ramp. The moment of inertia of a disk about its center is $I = MR^2/2$.

Calculate the acceleration of the disk.

Note that both the disk and the pulley have the same mass $M$. DO NOT assign them different masses.

*Hint:* Try to work neatly.
Problem 8. 25 points.

A merry-go-round in a playground spins with initial angular velocity $\omega_0$. You may assume the merry-go-round is a uniform disk of radius $R$ and mass $M$ with moment of inertia $I = MR^2/2$. A child, also of mass $M$, stands at the outer edge of the merry-go-round. The child then walks in until she gets to the center of the merry-go-round.

(a) (15 points) Calculate the angular velocity of the merry-go-round after the child reaches the center.

(b) (10 points) Calculate the work done by the child.
Potentially useful equations

Calculus:

Derivatives: If $x(t) = C t^n$ then
\[ \frac{dx}{dt} = C n t^{n-1} \]

Integrals:
\[ \int_{t_1}^{t_2} C t^n \, dt = C \left[ \frac{t_2^{n+1}}{n+1} - \frac{t_1^{n+1}}{n+1} \right] \]