Problem 1a

\[ v(t) = v_0 + \int_0^t a(t) \, dt = v_0 + \int_0^t (A - B t^2) \, dt \]
\[ = v_0 + At - \frac{1}{3} B t^3 \]

\[ x(t) = x_0 + \int_0^t v(t) \, dt = x_0 + \int_0^t \left( v_0 + At - \frac{1}{3} B t^3 \right) \, dt \]
\[ = x_0 + v_0 t + \frac{1}{2} At^2 - \frac{1}{12} B t^4 \]

Problem 1b

We want the max height. That occurs when \( v = 0 \). So we find the time that \( v \) is zero, and plug that time into our formula for position.

\[ v(t) = At - \frac{1}{3} B t^3 = 0 \quad \Rightarrow \quad t = \sqrt{\frac{3A}{B}} \]

So, then the position at that time is:

\[ x_{\text{max}} = x(t = \sqrt{\frac{3A}{B}}) = \frac{1}{2} A \frac{3A}{B} - \frac{1}{12} B \frac{9A^2}{B^2} = \frac{3A^2}{4B} \]

Problem 2

(a) \( a = dv/dt \) so we want the slope of the line at \( t = 6 \) s. The slope is the rise over the run: \( a = (0 \text{ m/s} - 20 \text{ m/s})/(6 \text{ s} - 2 \text{ s}) = -5 \text{ m/s}^2 \).

(b) \( x = x_0 + \int_0^t v(t) \, dt \). We can just take \( x_0 = 0 \) since we only want the total distance covered. So we just need the integral, or the area under the curve. This is a rectangle plus a triangle, or \( x = (20 \text{ m/s} \times 2 \text{ s}) + (20 \text{ m/s} \times 4 \text{ s})/2 = 80 \text{ m} \).

Problem 3 We want the Force, so we will use \( F = ma \). First, let’s get \( a \)

\[ \vec{r} = (x_0 + At^2) \hat{i} + (y_0 + Bt + \frac{1}{2} At^2) \hat{j} \]
\[ \vec{v} = \frac{d\vec{r}}{dt} = 2At \hat{i} + (B + At) \hat{j} \]
\[ \vec{a} = \frac{d\vec{v}}{dt} = 2A \hat{i} + A \hat{j} \]

Now, we want the magnitude, so \( a = \sqrt{a_x^2 + a_y^2} = \sqrt{5A} \). Finally, we use \( F = ma \), so

\[ F = \sqrt{5mA} \]
Problem 4
First note that $v_{0x} = v_0 \cos \theta$ and $v_{0y} = v_0 \sin \theta$. Now:

$$x = x_0 + v_{0x} t \quad \Rightarrow \quad R = 0 + v_0 \cos \theta t$$

and

$$y = y_0 + v_{0y} t - \frac{1}{2}gt^2 \quad \Rightarrow \quad H = 0 + v_0 \sin \theta t - \frac{1}{2}gt^2.$$ 

Solve the first for $t$ and substitute into the second:

$$t = \frac{R}{v_0 \cos \theta} \quad \Rightarrow \quad H = 0 + v_0 \sin \theta \frac{R}{v_0 \cos \theta} - \frac{1}{2}g \left( \frac{R}{v_0 \cos \theta} \right)^2$$

Solving for $v_0$

$$\frac{gR^2}{2v_0^2 \cos^2 \theta} = R \tan \theta - H$$

So

$$v_0 = \sqrt{\frac{gR^2}{2 \cos^2 \theta (R \tan \theta - H)}}$$

Problem 5
First, we draw the Free Body diagrams:

I’ve already labelled the $+x$ axis in the direction of acceleration. Next we write Newton’s second law for the $x$–direction for each block:

$$T - mg \sin \theta = ma$$

$$P - T - mg \sin \theta = ma$$

Adding these equations gives

$$a = \frac{P}{2m} - g \sin \theta$$

Substituting this into the first equation above gives

$$T = mg \sin \theta + ma = mg \sin \theta + \frac{P}{2} - mg \sin \theta$$

$$= \frac{P}{2}$$