Physics 218: Mechanics, Homework E1

Problem 1.

A particle moves in 1 dimension, and experiences a single conservative force given by

\[ F = -kx + bx^2 \]

where \( k \) and \( b \) are both positive constants.

(a) Calculate two positions (values of \( x \)) where the force is equal to zero.

(b) Derive the functional form of the potential energy of the particle, assuming that \( U(x = 0) = 0 \).

(c) The function that you should have gotten in part (b) is plotted below. This function has a local minimum at \( x = 0 \) with \( U(0) = 0 \) and a local maximum at \( x = x_1 \) with \( U(x_1) = U_1 \). Derive expressions for \( x_1 \) and \( U_1 \) in terms of the constants \( k \) and \( b \).

(d) Describe in words the motion of the particle for the following cases:

1. \( E = 0 \quad x = 0 \)
2. \( E = U_1 \quad x = x_1 \)
3. \( E < U_1 \quad 0 < x < x_1 \)
4. \( E < U_1 \quad x > x_1 \)
Problem 2.

A particle moves in 1 dimension, and experiences a single conservative force given by

\[ F = -4bx^3 + 2kx \]

where \( b \) and \( k \) are both positive constants.

(a) Calculate all positions (values of \( x \)) where the force is equal to zero.

(b) Derive the functional form of the potential energy of the particle, assuming that \( U(x = 0) = 0 \).

(c) The function that you should have gotten in part (b) is plotted below. This function has a local maximum at \( x = 0 \) with \( U(0) = 0 \) and local minima at \( x = \pm x_1 \) with \( U(x_1) = U_1 \). Derive expressions for \( x_1 \) and \( U_1 \) in terms of the constants \( k \) and \( b \).

(d) Describe in words the motion of the particle for the following cases:

1. \( E = 0 \quad x = 0 \)
2. \( E = U_1 \quad x = x_1 \)
3. \( 0 > E > U_1 \quad 0 < x < x_1 \)
4. \( E > 0 \)
Problem 3.

A particle moving in one dimension experiences a single conservative force given by

\[ F(x) = A - Bx \]

where \( A \) and \( B \) are positive constants.

(a) Calculate the potential energy function \( U(x) \) for this particle. Choose the potential energy to be zero at the position where the force is equal to zero.

(b) Make a sketch of this function. Indicate any stable or unstable equilibria.

(c) If the particle is released from rest at any position, where during its subsequent motion will it have maximum velocity?
A particle moves in 1 dimension, and experiences only conservative forces. The potential energy can be given by the following equation, which is plotted for your convenience:

\[ U(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2. \]

(a) Calculate the net force acting on the particle.

(b) Calculate all positions (values of \( x \)) where the force is equal to zero. For each of these positions, indicate whether the equilibrium is stable or unstable.

(c) Indicate for what values of \( x \) the force is in the +\( x \) direction. \((Hint: \) this is easy.)

(d) Describe in words the subsequent motion of the particle for the following conditions:

1. \( E = 0 \quad x = 0 \)
2. \( E = -1 \quad x = 1 \)
3. \( E = -8/3 \)

(e) The particle is released from rest at \( x = 3 \). Describe its subsequent motion, and calculate at which position the particle reaches its maximum velocity.
A particle of mass $m$ moves in 1 dimension under the influence of only conservative forces. The potential energy is given by

$$U(x) = \begin{cases} 
U_0 \left( \left( \frac{x}{a} \right)^2 - 4 \right) & x \leq 0 \\
U_0 \left( - \left( \frac{x}{a} \right)^3 + 3 \left( \frac{x}{a} \right)^2 - 4 \right) & 0 \leq x \leq 2a \\
0 & x \geq 2a
\end{cases}$$

where $U_0$ and $a$ are positive constants. This function is plotted below. The particle is initially at position $x_0 = 5a$, and has velocity $v_0 = -\sqrt{8U_0/m}$. (The minus sign means the particle is initially moving toward the left.)

(a) Calculate the total mechanical energy of the particle, and indicate this by drawing a constant energy line on the graph.

(b) What is the velocity of the particle when it reaches $x = 3a$?

(c) Calculate the magnitude and direction of the net force on the particle when it reaches the point $x = a$.

(d) Calculate the velocity of the particle when it reaches $x = 0$. 
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Problem 6.

A particle moves in 1 dimension, and experiences only conservative forces. The potential energy is given by the equation

\[ U(x) = U_0 \left( -\left( \frac{x}{a} \right)^3 + 3 \left( \frac{x}{a} \right) + 2 \right) \]

where \( U_0 \) and \( a \) are constants. This function is plotted here for your convenience:

(a) Calculate the net force acting on the particle.

(b) Calculate all positions (values of \( x \)) where the force is equal to zero. For each of these positions, indicate whether the equilibrium is stable or unstable.

(c) Suppose the particle is at position \( x = +2a \) and has velocity \( v = -\sqrt{10U_0/m} \). (The negative sign means the particle is moving in the negative \( x \) direction.) Calculate the velocity of the particle when it is at

1. \( x = a \)
2. \( x = 0 \)
3. \( x = -a \)
4. \( x = -2a \)

(d) The particle is released from rest at \( x = -3a \). Describe its subsequent motion, and state at which position the particle reaches its maximum velocity.