Physics 218: Mechanics. Exam 3, Secs. 813–815 22 Nov. 2005

Instructor: Dr. George R. Welch, 301D Doherty Building, 845-1571

Print your name **neatly**:

Last name: | | | | | | | | | | | | | | | | | |

First name: | | | | | | | | | | | | | | | | | |

Sign your name: ______________________________

Please fill in your Student ID number (UIN): __ __ __ – __ __ – __ __ __ __

IMPORTANT

Read these directions carefully:

- There are 6 problems totalling 100 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.

- **Indicate what you are doing!** We cannot give full credit for merely writing down the answer. **Neatness counts!** I will give generous partial credit **if** I can tell that you are on the right track. This means you must be *neat* and organized.

- Each problem with its associated figure is self explanatory. If you *must* ask a question, then come to the front, being as discrete as possible so as not to disturb others.

- Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.

Print your name: ____

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Problem 1. 15 points.

You are given expressions for the polar unit vectors in terms of the Cartesian unit vectors:

$$\begin{aligned} \hat{r} &= \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath} \\ \hat{\theta} &= -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath} \end{aligned}$$

Start with the position vector $\vec{r} = r\,\hat{r}$ and calculate the acceleration vector $\vec{a}$ expressed in *polar* coordinates.

You may need the derivatives:

$$\frac{d}{d\theta}\cos\theta = -\sin\theta \quad \text{and} \quad \frac{d}{d\theta}\sin\theta = \cos\theta\ .$$

Show all the steps in the derivation — do not use any other results that have been shown in class.

Don't forget to be neat.

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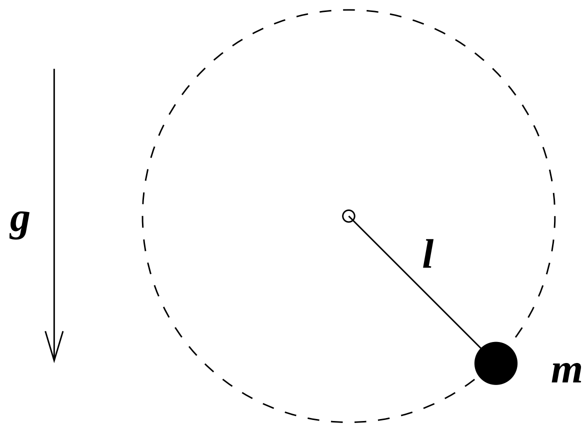
Problem 2. 15 points.

A particle of mass m is tied to a light strong rope of length l. The particle is spinning about the end of the rope in a vertical plane as shown:



The speed of the mass m is not constant. It speeds up and slows down as it goes around, just under the influence of gravity.

The tension in the rope when the particle is at the lowest point is given to be T_b. Calculate the tension in the rope when the particle reaches the highest point of the circle. **You must show your work to get credit**.

Hint: The particle is undergoing circular motion. The answer depends only on T_b and the weight of the particle.

Show your steps and **neatly** indicate what you are doing. There will be no credit for just writing down the answer.

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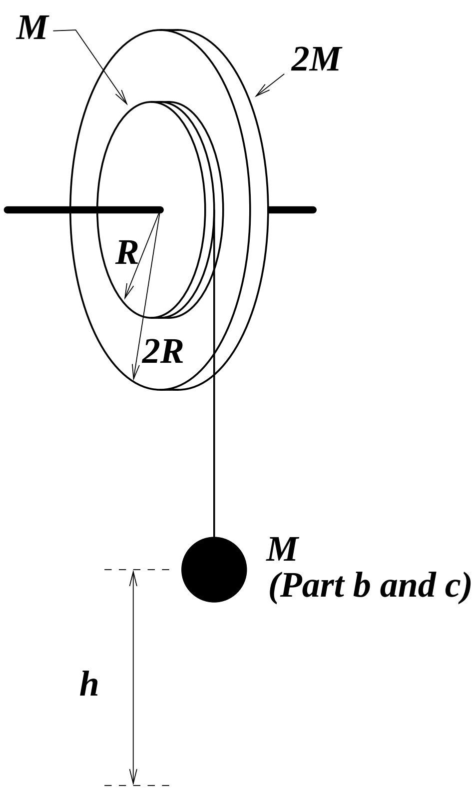
Problem 3. 20 points.

Two metal disks, one with radius R and mass M and the other with radius $2R$ and mass $2M$, are welded together and mounted on a frictionless axle through their common center to form a single pulley as shown in the figure.



(a) (4 points) Calculate the moment of inertia of this pulley. Express your answer in terms of M, R, and numerical factors. The moment of inertia of a disk about its center is $I = MR^2/2$.

(b) (11 points) A light string is wrapped around the edge of the smaller disk and a block, also of mass M, is suspended from the free end of the string. What is the velocity of the block after it has fallen through a distance h?

(c) (5 points) Same question as (b), but assume that the rope is wrapped around the larger disk.

Hint: In the answers to both parts (b) and (c), the mass and radius cancel out, so the answer depends only on g and h.

Make sure you are being neat. Working neatly will help you get it right.

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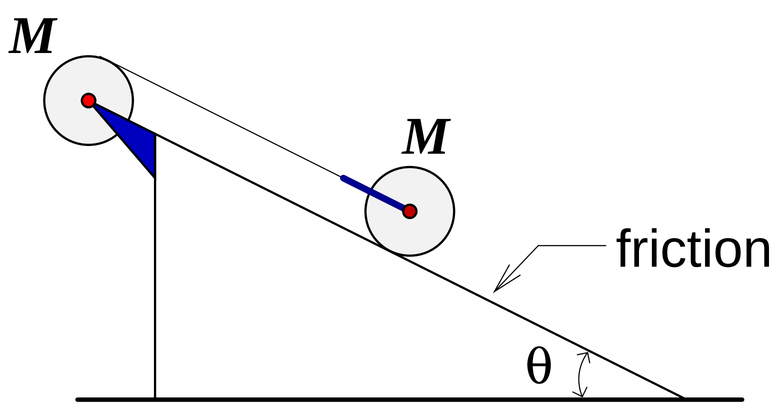
Problem 4. (20 points)

A solid disk of mass M and radius R is fixed at the top of a ramp. The ramp makes an angle θ with the horizontal. A rope is wrapped around the disk, and then attached to the axle of another solid disk of mass M and radius R. This second disk is allowed to roll without slipping down the ramp. The moment of inertia of a disk about its center is $I = MR^2/2$.

Calculate the magnitude of the linear acceleration of the center of the rolling disk.

Note that both the disk and the pulley have the same mass M. **DO NOT** assign them different masses.



If you work **neatly** I will find more partial credit for you!

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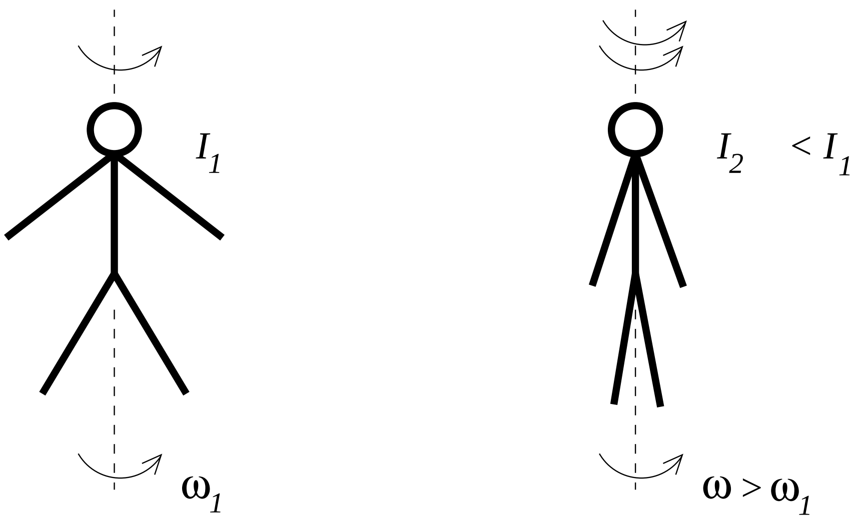
Problem 5. (15 points)

An ice skater goes into a spin. Assume that the ice is frictionless, and that there is no external torque on the skater.

Assume that the skater starts with arms out, and moment of inertia I_1 and angular velocity ω_1. The skater then brings in her arms and legs, and the moment of intertia decreases to I_2 which is less than I_1.

Calculate the work done by the skater.

Express your answer only in terms of I_1, I_2, ω_1, and numerical factors.



Did you work neatly? Neatness is important.

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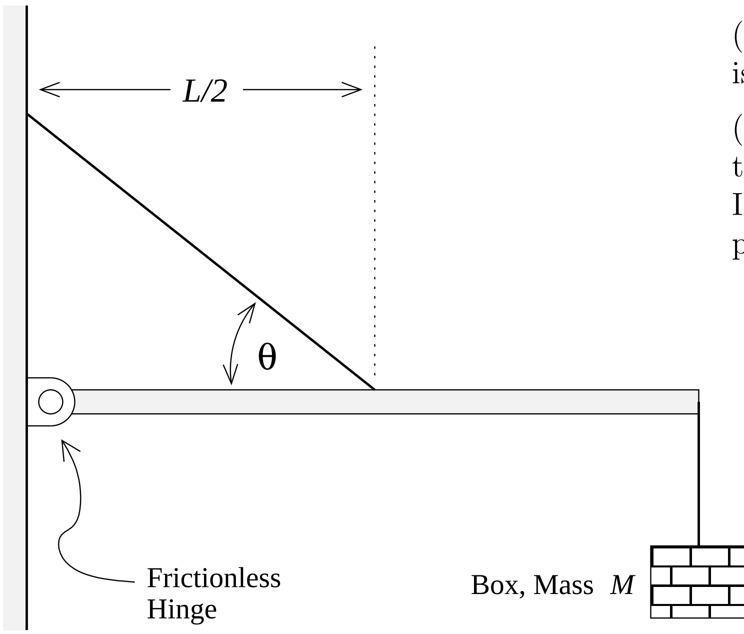
Problem 6. (15 points)

A rod of length L is attached to a vertical wall with frictionless hinge. The rod is light and strong – you can ignore its mass. At one end a large mass M is suspended from the rod. At the middle of the rod a rope is tied, and the other end of the rope is connected to the wall, such that the rod is held horizontal. In this configuration, the rope makes an angle θ with the rod.



(a) Calculate the tension in the rope that is connected to the wall.

(b) Calculate the x and y components of the force exerted on the rod by the hinge. Is the vertical component of this force pointing up or down?

You need to work neatly! Don't forget to be neat.

You may remove this sheet.

If you do remove this sheet,
DO NOT TURN IT IN!

Potentially useful equations

Calculus:

Derivatives:

$$\text{If } x(t) = C\,t^n \quad \text{then} \quad \frac{dx}{dt} = C\,n\,t^{n-1}$$

Integrals:

$$\int_{t_1}^{t_2} C\,t^n\,dt = C\left[\frac{t_2^{n+1}}{n+1} - \frac{t_1^{n+1}}{n+1}\right]$$

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