

Print your name **neatly**:

Last name:

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First name:

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Sign your name: _____

Please fill in your Student ID number (UIN): _____-_____-_____

IMPORTANT

Read these directions carefully:

- There are 8 problems totalling 200 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.
- **Indicate what you are doing!** We cannot give full credit for merely writing down the answer. **Neatness counts!** I will give generous partial credit **if** I can tell that you are on the right track. This means you must be *neat* and organized.
- Each problem with its associated figure is self explanatory. If you *must* ask a question, then come to the front, being as discrete as possible so as not to disturb others.
- Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.

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Physics 218: Mechanics, Fall, 2005, Final Exam

Problem 1. 25 points.

A bird takes off from the ground, initially moving both forward (the x -direction) and upward (the y -direction). As the bird flies, it smoothly reverses its x motion, while still moving upward. Its velocity as a function of time can be written as

$$\vec{v}(t) = (\alpha - \beta t^2)\hat{i} + \gamma t\hat{j}$$

where α , β , and γ are given positive constants. Take the origin of your coordinate system to be the initial position of the bird ($x_0 = 0$ and $y_0 = 0$).

- (a) Calculate both the acceleration and position vectors as functions of time.
- (b) Calculate the height of the bird when it reaches its maximum x coordinate.

Don't forget to be neat.

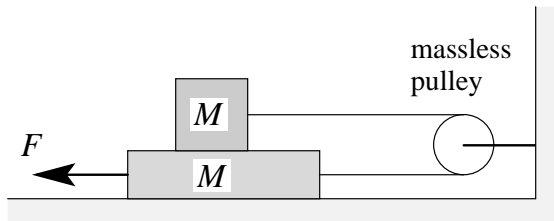
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Physics 218: Mechanics, Fall, 2005, Final Exam

Problem 2. 25 points.

A block of mass M rests on a rough horizontal surface. Another block, also of mass M is sitting on top of the first block. The blocks are connected by a taut strong light rope that passes around a massless pulley so that when the bottom block moves to the left then the top block must move to the right. The coefficient of kinetic friction between all surfaces is μ .

A constant horizontal force F is applied to the bottom block as shown. Suppose that this force is large enough to cause the blocks to move. Calculate the acceleration of the blocks.



Remember, both blocks have the same mass. Do not assign the two blocks different masses.

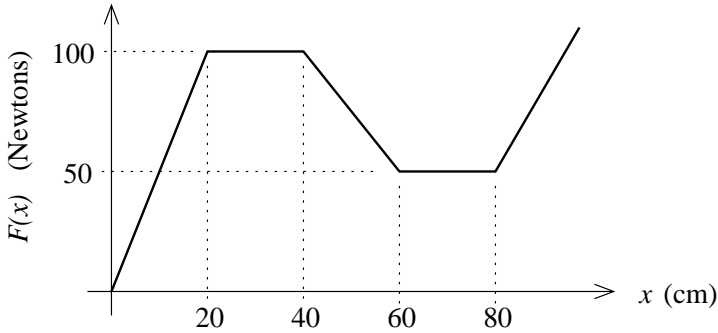
You still need to be **neat**. Neatness still helps.

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Physics 218: Mechanics, Fall, 2005, Final Exam

Problem 3. 25 points.

You are designing a compound bow (as in bow and arrow) with a long pull. The bow is fixed in place, and the force required to pull back the string is plotted below as a function of the distance by which the string has been pulled.



Suppose an arrow of mass 11 grams is fitted to the bow, pulled back 80 cm, and released. Assume that the force exerted on the arrow as it is pulled by the string is the same as the force plotted above. What will be the velocity of the arrow when it leaves the bow?

Note: I have cooked the numbers so that the answer is a nice even number. You do not need a calculator if you can divide 2 by 2 and add simple numbers.

Note 2: You must *indicate what you are doing!* It may be easy to goof the numbers, and unless you indicate what you are doing you won't get any partial credit.

Note 3: Pay attention to the units!

Make sure you are being neat. Working neatly will help you get it right.

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Problem 4. (25 points)

A particle of mass m moves in 1-dimension. The total force on the particle is given by

$$F(x) = -kx + \alpha x^3$$

where k and α are given positive constants.

(a) Calculate a potential energy function for this particle assuming that the potential energy is zero at the origin.

(b) When the particle is at position $x = +\sqrt{k/\alpha}$ it is observed to have velocity $v = -\sqrt{k^2/(4m\alpha)}$. Calculate the speed of the particle when it reaches position $x = \sqrt{k/(2\alpha)}$. Express the answer in terms of k , α , m , and numerical factors.

If you work **neatly** I will find more partial credit for you!

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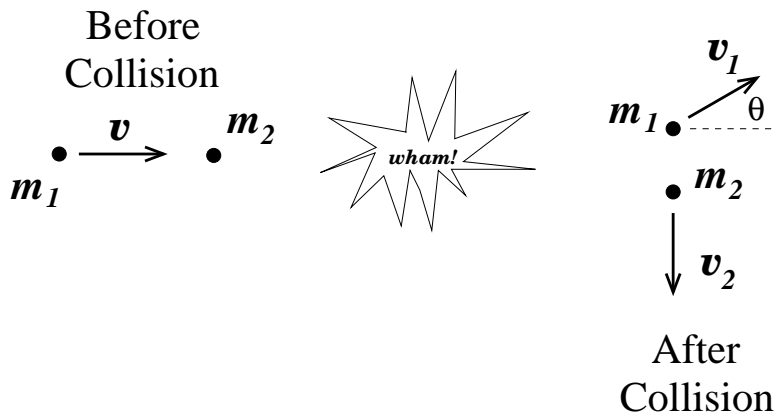
Physics 218: Mechanics, Fall, 2005, Final Exam

Problem 5. (25 points)

Two balls, with masses m_1 and m_2 , move without friction in a horizontal plane. Ball 1 moves in the $+x$ -direction with speed v and ball 2 is stationary at the origin. The balls collide inelastically.

After the collision, ball 1 moves at an angle θ from the x -axis, and ball 2 moves in the $-y$ -direction

Calculate the speeds of both balls after the collision in terms of m_1 , m_2 , v , and θ .



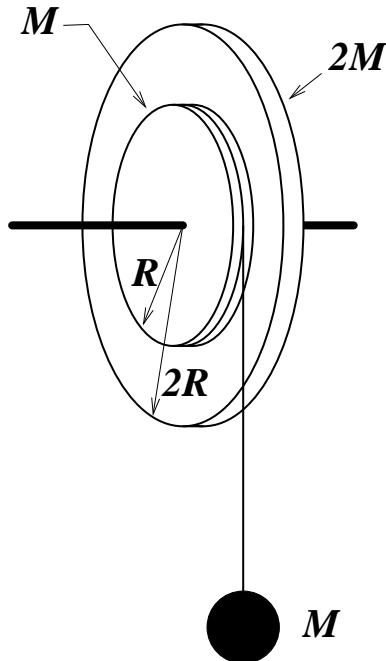
Did you work neatly? Neatness is important.

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Problem 6. (25 points)

Two metal disks, one with radius R and mass M and the other with radius $2R$ and mass $2M$, are welded together and mounted on a frictionless axle through their common center to form a single pulley as shown in the figure.



(a) (5 points) Calculate the moment of inertia of this pulley. Express your answer in terms of M , R , and numerical factors. The moment of inertia of a disk about its center is $I = MR^2/2$.

(b) (15 points) A light string is wrapped around the edge of the smaller disk and a block, also of mass M , is suspended from the free end of the string. What is the magnitude of the linear acceleration of the block.

(c) (5 points) Same question as (b), but assume that the rope is wrapped around the larger disk.

Hint: In the answers to both parts (b) and (c), the mass and radius cancel out, so the answer depends only on g and numerical factors.

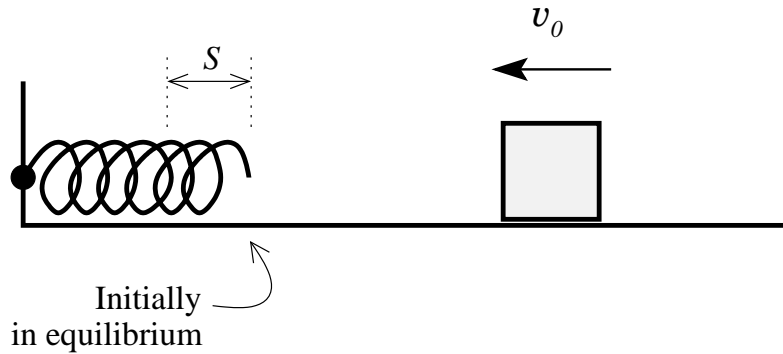
Make sure you are being neat. Working neatly will help you get it right.

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Problem 7. (25 points)

A block is moving at speed v_0 along a frictionless horizontal surface toward a spring. The spring has negligible mass. The block hits the spring, compressing it a maximum distance S , then bounces off and moves in the opposite direction with the same speed it started (because there is no friction).



Calculate how much time the block will be in contact with the spring. Express your answer in terms of v_0 and S . The answer *does not* depend on the mass of the block.

Hint: Assume the block has mass M . Calculate the spring constant. Then cancel the mass when you get the answer.

Always work neatly. Indicate what you are doing to get partial credit.

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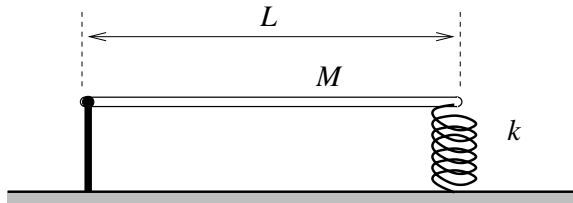
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Problem 8. (25 points)

A long rod of mass M and length L is pivoted at one end. The other end of the rod is connected to a spring of force constant k which is rigidly mounted as shown. The rod is displaced from its equilibrium position by a *small* angle and released. Calculate the period of small oscillations of the rod about its equilibrium position.

Ignore gravity: only consider the effect of the spring on the rod.

The moment of inertia of a long thin rod of mass M and length L rotating about *its center* is $I = ML^2/12$. You need the moment of inertia of the rod rotating about *its end*.



Show your steps and **neatly** indicate what you are doing. There will be no credit for just writing down the answer.

You may remove this sheet.

If you do remove this sheet,
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Potentially useful equations

Calculus:

Derivatives:

$$\text{If } x(t) = C t^n \quad \text{then} \quad \frac{dx}{dt} = C n t^{n-1}$$

Integrals:

$$\int_{t_1}^{t_2} C t^n dt = C \left[\frac{t_2^{n+1}}{n+1} - \frac{t_1^{n+1}}{n+1} \right]$$

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