Physics 218 — Fall 2005 — Exam 1 Solution **George R. Welch**

Problem 1 (This was *identical* to supplementary homework 1, problem 3. See also assigned book problems 2-9b and 2-18b.)

$$\begin{aligned} v &= \frac{dx}{dt} = \underline{\underline{\alpha - \frac{1}{2}\,\beta\, t^{-\frac{1}{2}}}} \\ a &= \frac{dv}{dt} = \underline{\underline{+\frac{1}{4}\,\beta\, t^{-\frac{3}{2}}}} \end{aligned}$$

Problem 2 (This was similar to supplementary homework 2, problem 8. See also assigned book problems 2-20a and 2-51b.)

We are given the acceleration, so first let's calculate the velocity and position functions:

$$\begin{aligned} v(t) &= v_0 + \int_0^t a(t)\,dt = v_0 + \int_0^t (-kt)\,dt = v_0 - \frac{1}{2}kt^2 \\ x(t) &= x_0 + \int_0^t v(t)\,dt = 0 + \int_0^t (v_0 - \frac{1}{2}kt^2)\,dt = v_0\,t - \frac{1}{6}kt^3 \end{aligned}$$

(b) The maximum position occurs when then velocity is zero. So, we solve for the time when the velocity is zero, and substitute that into the expression for $x(t)$

$$v_0 - \frac{1}{2}kt^2 = 0 \quad \Longrightarrow \quad t = \sqrt{\frac{2v_0}{k}}$$

$$x_{\text{max}} = x\left(t = \sqrt{\frac{2v_0}{k}}\right) = v_0\sqrt{\frac{2v_0}{k}} - \frac{1}{6}k\left(\sqrt{\frac{2v_0}{k}}\right)^3 = v_0\sqrt{\frac{2v_0}{k}} - \frac{1}{6}k\,\frac{2v_0}{k}\sqrt{\frac{2v_0}{k}} = \underline{\underline{\frac{2}{3}v_0\sqrt{\frac{2v_0}{k}}}}$$

(a) The maximum velocity occurs when the acceleration is zero. We are given $a(t) = -kt$ so it is clear this is zero when $t = 0$. Thus $\underline{\underline{v_{\text{max}} = v_0}}$.

(c) The position at $t = 0$ is just $\underline{\underline{x = 0}}$.

Problem 3 (This was similar to supplementary homework 2, problem 1. See also assigned book problem 53a.)

We are given a graph of $v(t)$. The instantaneous acceleration at any time is just the slope of this graph, and the distance covered is the intergral from $t = 0$, which is just the area under the curve.

(a) At $t = 25$ s the slope is $\underline{\underline{a = 0}}$. The area under the curve is just the area of the triangle that goes from 20 m/s at $t = 0$ to 0 at $t = 20$. Thus,

$$x = \frac{1}{2} \times 20\,\frac{\text{m}}{\text{s}} \times 20\,\text{s} = \underline{\underline{200\text{ m}}}$$

(b) At $t = 40$ s the slope is negative. We look at the line segment from $t = 30$ s to $t = 40$ s

$$a = \frac{-10\text{ m/s}}{10\text{ s}} = \underline{\underline{-1\text{ m/s}^2}}$$

Finally, the distance covered at $t = 40$ s is the area under the curve. This is the (positive) area computed in part (a) minus the area of the triangle from $t = 30$ s to $t = 40$ s

$$x = 200 - \frac{1}{2} \times 10\,\frac{\text{m}}{\text{s}} \times 10\,\text{s} = \underline{\underline{150\text{ m}}}$$

Problem 4 (This was just 2-d kinematics like all the problems in supplementary homework 5. See also assigned book problem 3-44.)

The problem is asking us how high the car is and the vertical component of its velocity at the moment the rocket engine turns off. So, first let's find out how long the rocket is on.

$$v_x = v_{x0} + \int_0^t a_x(t)\,dt = v_0 + \frac{1}{2}bt^2$$

At the moment the engine turns off, we are given $v_x = 2v_0$. Call this time t_E, so

$$2v_0 = v_0 + \frac{1}{2}bt_E^2 \quad\Longrightarrow\quad t_E = \sqrt{\frac{2v_0}{b}}$$

Now, we are given the vertical component of the acceleration during this time, so lets calculate formulas for the vertical component of the velocity and position:

$$\begin{aligned} v_y(t) &= v_{y0} + \int_0^t a_y(t)\,dt = 0 + \int_0^t ct^2\,dt = \frac{1}{3}ct^3 \\ y(t) &= y_0 + \int_0^t v_y(t)\,dt = 0 + \int_0^t (\frac{1}{3}ct^3)\,dt = \frac{1}{12}ct^4 \end{aligned}$$

Now we just substitute the time the motor turns off into these formulas:

$$\text{(a) } y(t_E) = \frac{1}{12}c\left(\sqrt{\frac{2v_0}{b}}\right)^4 = \underline{\underline{\frac{1}{3}c\frac{v_0^2}{b^2}}} \qquad \text{(b) } v_y(t_E) = \frac{1}{3}c\left(t = \sqrt{\frac{2v_0}{b}}\right)^3 = \underline{\underline{\frac{2}{3}\frac{cv_0}{b}\sqrt{\frac{2v_0}{b}}}}$$

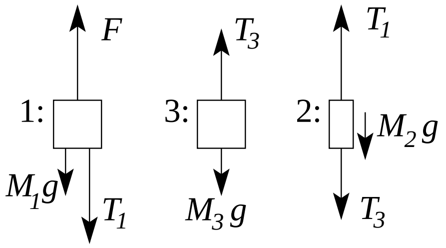
Problem 5 (This was similar to supplementary homework 4. See also assigned book problem 1-48c)

$$\begin{aligned} \text{(a)}\quad |\vec{\mathbf{A}}+\vec{\mathbf{B}}| &= |(5-4)\hat{\imath} + (-4+5)\hat{\jmath}| = |\hat{\imath}+\hat{\jmath}| = \sqrt{1^2+1^2} = \underline{\underline{\sqrt{2}}} \\ |\vec{\mathbf{A}}-\vec{\mathbf{B}}| &= |(5+4)\hat{\imath} + (-4-5)\hat{\jmath}| = |9\hat{\imath}+9\hat{\jmath}| = \sqrt{9^2+9^2} = \sqrt{162} = \underline{\underline{9\sqrt{2}}} \end{aligned}$$

(b) $\underline{\underline{\text{No}}}$ because $|\vec{\mathbf{A}}+\vec{\mathbf{B}}| \neq 1$. (c) $c|\vec{\mathbf{A}}+\vec{\mathbf{B}}| = 1 \Longrightarrow c\sqrt{2} = 1 \Longrightarrow \underline{\underline{c = 1/\sqrt{2}}}$

Problem 6 (This was *identical* to assigned book problem 4-49. I also worked it in class.)

First we show the Free-body diagrams. You only had to draw the first two, but I am showing all three here:



From these diagrams, we can write down Newton's second law of motion for each of the three bodies:

$$\begin{aligned} F - M_1 g - T_1 &= M_1 a \\ T_3 - M_3 g &= M_3 a \\ T_1 - M_2 g - T_3 &= M_2 a \end{aligned}$$

We can easily solve for a by adding all three equations:

$$F - (M_1+M_2+M_3)g = (M_1+M_2+M_3)a \quad\Longrightarrow\quad a = \frac{F}{M_1+M_2+M_3} - g$$

and we can substitute this into the first equation and solve for T_1. After simplifying the result, we find

$$\underline{\underline{T_1 = \frac{M_2+M_3}{M_1+M_2+M_3}F}}$$