Problem 1 (This was very similar to assigned book problems 5-62. It was not very different from 5-81. I also did something like this in class.)

First let’s draw the free-body diagrams for all three blocks. This is the same for parts (a) and (b)

\[ T_1 M_3 g \]
\[ \vec{T}_1 \]
\[ \vec{f}_2 \]
\[ N_1 \]
\[ M g \]
\[ f_2 \]
\[ N_2 \]
\[ f_1 \]
\[ +x \]
\[ \nabla \]

Now we write Newton’s second law for blocks 2 and 3.

\[ M_3 g - T_1 = M_3 a \]
and
\[ T_1 - f_1 - f_2 = M a \]

(a) First, \( a = 0 \) so \( T_1 = M_3 g \). Second, we are looking for the maximum value of \( M_3 \) when the friction forces are maximum. The maximum values of the friction is \( \mu_s N \), so we have \( f_1 = \mu_s N_1 = 2 \mu_s M g \) and \( f_2 = \mu_s N_2 = \mu_s M g \). Putting this together,

\[ M_3 g - 2 \mu_s M g - \mu_s M g = 0 \implies M_3 = 3 \mu_s M \]

(b) Here we have \( M_3 = 3M \) and \( f = \mu_k N \) so the equations become

\[ 3M g - T_1 = 3 M a \]
and
\[ T_1 - 2 \mu_k M g - \mu_k M g = M a \]

This is 2 equations in 2 unknowns. We can add them and find \( 3M g - 3 \mu_k M g = 4 M a \) so
\[ a = 3(1 - \mu_k)g/4. \]

Problem 2 (This was identical to assigned book problems 6-31 (with only the numbers changed). It was also given as a quiz in your recitation.)

We use the Work-Energy theorem: \( W = K_f - K_i \). We recall that \( W = \int_{x_i}^{x_f} F(x) \, dx \). We are given a graph of \( F(x) \) so the Work is just the area under the curve: \( W = (12 \times 6)/2 + (6 \times 6)/2 = 54 \) joules. Now, we know the sled starts from rest, so \( K_i = 0 \). Thus,

\[ 54 = \frac{1}{2} M v^2 = \frac{3}{2} v^2 \implies v = 6 \text{ m/s} \]

Problem 3 (Part (a) is identical to assigned book problem 6.81. Part (b) is just Newton’s second law and Hooke’s law.)

(a) \( E_i = E_f \implies K_i + U_i = K_f + U_f \implies \frac{1}{2} M v_0^2 + 0 = 0 + \frac{1}{2} k x_f^2 \implies x_f = \sqrt{\frac{M}{k}} v_0 \)

(b) \( F = -kx = Ma \implies -k \sqrt{\frac{M v_0^2}{k}} = Ma \implies a = -\sqrt{\frac{k}{M}} v_0 \)
Problem 4 (This was identical to assigned book problem 7-67. Only the numbers were changed to protect the innocent.)

(a) \[ U(x) = -\int_0^x F(x) \, dx + 0 = -\int_0^x (-kx - \beta x^2) \, dx = \frac{1}{2} k x^2 + \frac{1}{3} \beta x^3 \]

(b) \[ E_i = E_f \quad \implies \quad K_i + U_i = K_f + U_f \quad \implies \quad 0 + U \left( \frac{k}{\beta} \right) = \frac{1}{2} m v_f^2 + U \left( \frac{k}{2\beta} \right) \]

\[ U \left( \frac{k}{\beta} \right) = \frac{1}{2} k \left( \frac{k}{\beta} \right)^2 + \frac{1}{3} \beta \left( \frac{k}{\beta} \right)^3 = \left( \frac{1}{2} + \frac{1}{3} \right) \frac{k^3}{\beta^2} = \frac{5}{6} \frac{k^3}{\beta^2} \]

\[ U \left( \frac{k}{2\beta} \right) = \frac{1}{2} k \left( \frac{k}{2\beta} \right)^2 + \frac{1}{3} \beta \left( \frac{k}{2\beta} \right)^3 = \left( \frac{1}{8} + \frac{1}{24} \right) \frac{k^3}{\beta^2} = \frac{1}{6} \frac{k^3}{\beta^2} \]

So \[ \frac{1}{2} m v_2 = \left( \frac{5}{6} - \frac{1}{6} \right) \frac{k^3}{\beta^2} \quad \implies \quad v = \sqrt{\frac{4k^3}{3m\beta^2}} \quad \text{putting in the numbers gives} \quad v = 2 \, \text{m/s} \]

Problem 5 (It was my hope that after the 6 supplemental problems on conservation of energy, that this would be pretty easy. It is also pretty similar to assigned book problem 7-86.)

All answers were false except (f) which was true. I took off 2 points for the first wrong answer, and three points for each subsequent wrong answer. (This was to prevent guessing.)

Problem 6 (Part (a) is identical to assigned book problem 8.70. Part (b) is pretty similar.)

(a) Conservation of momentum during the collision: \[ mv_0 = (M + m) V \quad \text{where} \quad V \quad \text{is the speed of the block after the collision.} \]

Conservation of energy after the collision: \[ \frac{1}{2} (M + m) V^2 = \frac{1}{2} k S_1^2 \]

So, \[ \frac{1}{2} (M + m) \left( \frac{mv_0}{M + m} \right)^2 = \frac{1}{2} k S_1^2 \quad \implies \quad v_0 = \frac{M + m}{m} \sqrt{\frac{k}{M + m}} S_1 \]

(b) Conservation of momentum during the collision: \[ mv_0 = M V \]

Conservation of energy after the collision: \[ \frac{1}{2} M V^2 = \frac{1}{2} k S_2^2 \]

So, \[ \frac{1}{2} M \left( \frac{mv_0}{M} \right)^2 = \frac{1}{2} k S_2^2 \quad \implies \quad v_0 = M \sqrt{\frac{k}{M}} S_2 \]