Physics 218: Mechanics. Final Exam, Secs. 810–812 12 Dec. 2006

Instructor: Dr. George R. Welch, 301D Doherty Building, 845-1571

Print your name **neatly**:

Last name: | | | | | | | | | | | | | | | | | |

First name: | | | | | | | | | | | | | | | | | |

Sign your name: ______________________________

Please fill in your Student ID number (UIN): __ __ __ –__ __ –__ __ __ __

IMPORTANT

Read these directions carefully:

- There are 8 problems totalling 200 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.

- **Indicate what you are doing!** We cannot give full credit for merely writing down the answer. **Neatness counts!** I will give generous partial credit **if** I can tell that you are on the right track. This means you must be *neat* and organized.

- Each problem with its associated figure is self explanatory. If you *must* ask a question, then come to the front, being as discrete as possible so as not to disturb others.

- Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.

Print your name: ______________________________

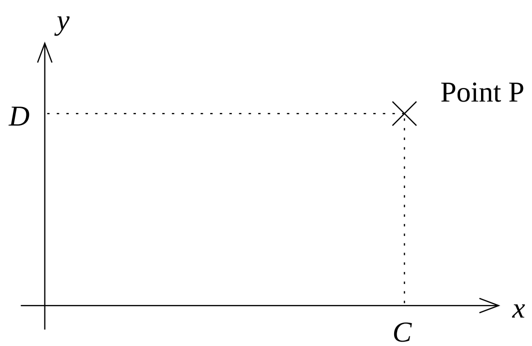
Physics 218: Mechanics, Fall, 2006, Final Exam

Problem 1. 25 points.

A robot moves on a horizontal plane. The robot is placed at the origin of the coordinate system shown below, and it starts to move. The robot has been programmed to accelerate as a function of time according to

$$\vec{a}(t) = bt\,\hat{\imath} - d\,\hat{\jmath}$$

where b and d are known constants. Your job is to program an initial velocity so that the robot will pass through the point P (x and y coordinates C and D) shown in the Figure.



(a) (15 points) Your team-mates have chosen the x-component of the initial velocity to be zero. Calculate what y-component of the initial velocity you should choose so that the robot reaches the point P.

(b) (10 points) Some time after the robot passes through the point P, it crosses the x-axis. Calculate the value of x when this happens.

Express your answers in terms of b, d, C, and D.

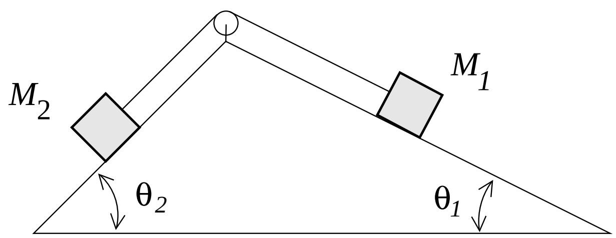
Don't forget to be neat.

Print your name: ______________________________

Physics 218: Mechanics, Fall, 2006, Final Exam

Problem 2. 25 points.

Consider two blocks of mass M_1 and M_2 placed on rough flat surfaces. Block M_1 is placed on a plane that makes an angle θ_1 with the horizontal, and block M_2 is placed on a plane that makes an angle θ_2 with the horizontal. The coefficient of friction between block M_1 and the surface is μ_1 and the coefficient of friction between block M_2 and the surface is μ_2 . The blocks are connected by a light (massless) unstretchable cable, which runs parallel to the surfaces and passes over a massless pulley.



Suppose that block M_1 is so heavy that it accelerates down the ramp.

(a) (10 points) Draw **neat** and clear free-body diagrams for each block. Indicate your coordinate system. Neatness will count as a significant part of your grade.

(b) (15 points) Calculate the acceleration of the masses.

You must be **neat.** Neatness helps.

Print your name: ______________________________

Physics 218: Mechanics, Fall, 2006, Final Exam

Problem 3. 25 points.

A designer in the department of Mechanical Engineering at Texas Atomic and Molecular University has devised a nonlinear spring that behaves differently according to whether it is being compressed or stretched. Suppose that you connect a mass to the end of the spring, and measure the force that the spring exerts on the mass. Call the axis along which you compress or stretch the spring the x-axis, and let $x = 0$ be the equilibrium point and $x > 0$ be stretching the spring. It is found that the force is giving by:

$$F(x) = \begin{cases} -k_1\, x & x \leq 0 \\ -k_2\, x^3 & x \geq 0 \end{cases}$$

Suppose the block is pushed (so that the spring is compressed) a distance D from equilibrium, and then released from rest. The spring pushes the mass back toward equilibrium, and then gets stretched as the mass continues past the equilibrium point.

Calculate how far the spring gets stretched.

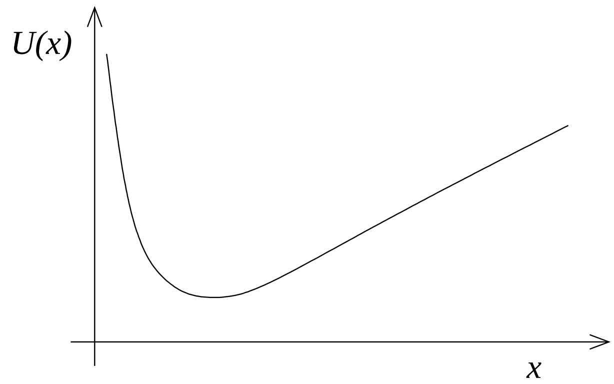
Express your answer in terms of k_1, k_2, and D. You do not need to know the mass of the block.

Make sure you are being neat. Working neatly will help you get it right.

Print your name: ______________________________

Physics 218: Mechanics, Fall, 2006, Final Exam

Problem 4. (25 points)



A particle of known mass M moves in 1 dimension under the influence of only conservative forces. The potential energy function for the particle is given by

$$U(x) = \frac{A}{x} + B\,x$$

where A and B are known positive constants. A plot of this function is shown in the Figure.

(a) (2 points) Indicate on the x-axis of the graph above all points where the net force on the particle is zero.

(b) (8 points) Derive an expression for the force on the particle as a function of position x.

(c) (10 points) Suppose that the total energy of the particle is $E = 4\sqrt{AB}$. Derive an equation for the positions at which the particle has zero velocity.

(d) (5 points) Suppose that the particle is at some position where the net force is equal to the constant B. (You can verify from the potential energy function that the constant B has dimensions of force.) What other piece of information do you need to calculate the total mechanical energy of the particle? For full credit, your answer must not exceed 1 word.

Did you work neatly? Neatness is important.

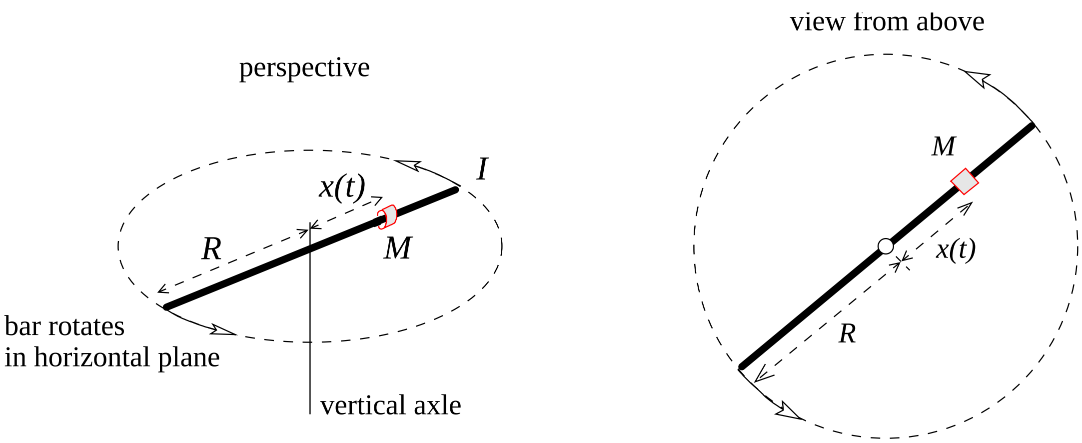
Print your name: ____

Physics 218: Mechanics, Fall, 2006, Final Exam

Problem 5. (25 points)

A horizontal bar is fixed to a vertical axle, so that the bar rotates about its center. The axle is mounted on frictionless bearings. The distance from the axle to either end of the bar is R (so the length of the bar is $2R$. The moment of inertia of the bar about the axle is I.

A heavy sleeve of mass M is mounted on the bar, and can move back and forth along the bar. (For example, it might have a motor inside, and move along the bar via gears that mesh into the bar.) We denote the distance of the sleeve from the axle $x(t)$. This is shown in the Figure:



Suppose that the sleeve starts at $t = 0$ at the outer end of the bar, and moves in toward the axle with constant radial velocity, so that

$$x(t) = R - v_r t$$

Suppose that at $t = 0$ when the sleeve is at the outer edge of the bar, that the angular velocity of the bar about the axle is ω_0.

(a) (10 points) Find an expression for the angular velocity of the bar about the axle as a function of time. Express your answer in terms of I, M, R, v_r, ω_0 , and, of course, t.

...Continued on the next page

If you work **neatly** I will find more partial credit for you!

Print your name: ____

Physics 218: Mechanics, Fall, 2006, Final Exam

Problem 5, *continued.*

(b) (10 points) Now suppose that the bar is much lighter than the sleeve, so that its moment of inertia is negligible. That is, take $I = 0$. Calculate the tangential component of the acceleration (a_θ) of the sleeve.

*H*int: The answer does *not* depend on any of the following: M, R, v_r, ω_0, or t.

(c) (5 points) Explain why the answer does not depend on M, R, v_r, ω_0, or t. Your answer should be short – no more than 2 or three sentences.

Organize your thoughts, and write clearly. Otherwise I won't give as many points.

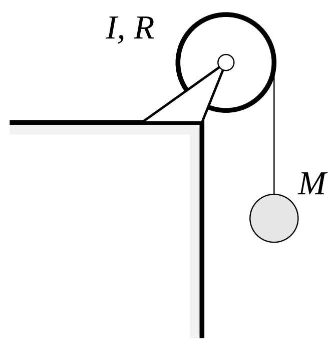
Print your name: ____

Physics 218: Mechanics, Fall, 2006, Final Exam

Problem 6. (25 points)

A block of mass M is hung from a massless, unstretchable, cable which is wrapped around a large heavy flywheel. The flywheel has moment of inertia I about its center and radius R. The cable does not slip on the flywheel, so that as the block goes down the flywheel spins.

Calculate the acceleration of the block and the tension in the cable.



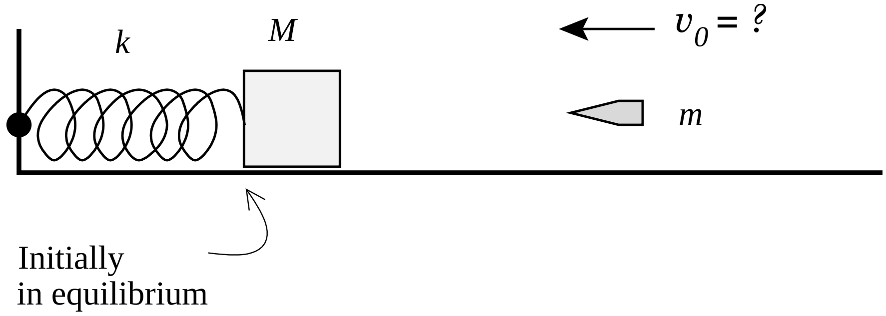
Make sure you are being neat. Working neatly will help you get it right.

Print your name: ____

Physics 218: Mechanics, Fall, 2006, Final Exam

Problem 7. (25 points)

A rifle bullet with mass m is fired toward a block with mass M that rests on a frictionless, horizontal surface and is attached to a coil spring of force constant k as shown in the Figure.



(a) (15 points) Suppose the bullet strikes and embeds itself in the block, and that the impact compresses the spring a distance S. Calculate the initial velocity of the block.

(b) (10 points) Calculate how long it takes from the moment the bullet strikes the block until the block reaches its maximum compression S.

Express your answers in terms of k, M, m, and S.

Always work neatly. Indicate what you are doing to get partial credit.

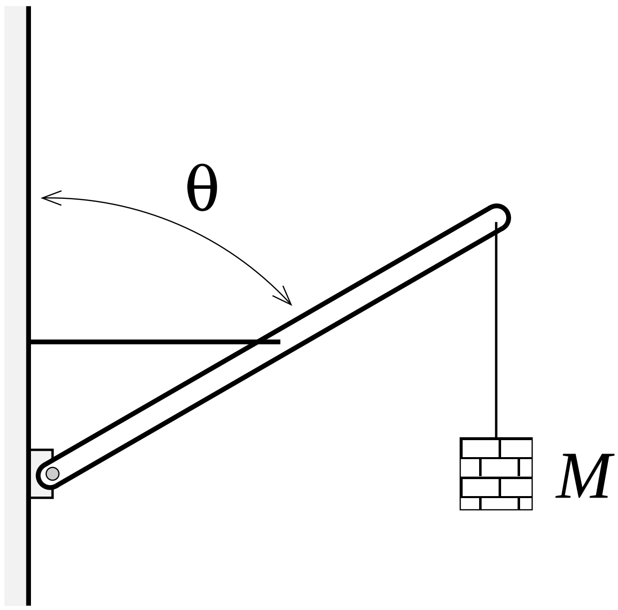
Print your name: ______________________________

Physics 218: Mechanics, Fall, 2006, Final Exam

Problem 8. (25 points)

A light strong rod (assume it is massless) is hinged to a vertical wall. Denote the length of the rod by L. A light strong cable is attached to the wall, and to the midpoint of the rod, such that the cable is horizontal, while the rod makes an angle θ with the wall.

A large weight of mass M is connected to the end of the rod as shown in the Figure.



Calculate the tension in the horizontal cable, and calculate the magnitude of the force that the hinge exerts on the rod.

Show your steps and **neatly** indicate what you are doing. There will be no credit for just writing down the answer.

You may remove this sheet.

If you do remove this sheet,
DO NOT TURN IT IN!

Potentially useful equations

Calculus:

Derivatives:

$$\text{If } x(t) = C\,t^n \quad \text{then} \quad \frac{dx}{dt} = C\,n\,t^{n-1}$$

Integrals:

$$\int_{t_1}^{t_2} C\,t^n\,dt = C\left[\frac{t_2^{n+1}}{n+1} - \frac{t_1^{n+1}}{n+1}\right]$$

Acceleration in polar coordinates:

$$\vec{a} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(2\omega\frac{dr}{dt} + r\alpha\right)\hat{\theta}$$

where

$$\omega = \frac{d\theta}{dt} \quad \text{and} \quad \alpha = \frac{d\omega}{dt}$$

If you do remove this sheet,
DO NOT TURN IT IN!
