Problem 1. (This is the whole point of chapter XII in the book. I did it in class. I told you it would be on the exam.)

$$\hat{r} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y} \qquad \hat{\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$$

$$\frac{d\hat{r}}{d\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y} = \hat{\theta} \qquad \frac{d\hat{\theta}}{d\theta} = -\cos\theta\,\hat{x} - \sin\theta\,\hat{y} = -\hat{r}$$

$$\frac{d\hat{r}}{dt} = \frac{d\hat{r}}{d\theta}\frac{d\theta}{dt} = \hat{\theta}\omega \qquad \frac{d\hat{\theta}}{dt} = \frac{d\hat{\theta}}{d\theta}\frac{d\theta}{dt} = -\hat{r}\omega$$

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} = \frac{d}{dt}r\,\hat{r} \\ &= \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt} \\ &= \frac{dr}{dt}\hat{r} + r\omega\,\hat{\theta} \end{aligned}$$

$$\begin{aligned} \vec{a} &= \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{dr}{dt}\hat{r} + r\omega\,\hat{\theta}\right) \\ &= \frac{d^2r}{dt^2}\hat{r} + \frac{dr}{dt}\frac{d\hat{r}}{dt} + \frac{dr}{dt}\omega\,\hat{\theta} + r\frac{d\omega}{dt}\hat{\theta} + r\omega\frac{d\hat{\theta}}{dt} \\ &= \frac{d^2r}{dt^2}\hat{r} + \frac{dr}{dt}\omega\,\hat{\theta} + \frac{dr}{dt}\omega\,\hat{\theta} + r\alpha\,\hat{\theta} - r\omega^2\,\hat{r} \\ &= \underline{\underline{\left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(2\omega\frac{dr}{dt} + r\alpha\right)\hat{\theta}}} \end{aligned}$$

Problem 2. (See problem 6 or exercise 3 in chapter X.)

We just use conservation of linear momentum: $\vec{P}_i = \vec{P}_f$ Note that this is a collision, and we don't know about the forces involved, so we cannot say if energy is conserved. We resolve the conservation of momentum equation in rectangular coordinates:

$$P_{xi} = P_{xf} \quad\Longrightarrow\quad 3mv_3 - mv_1\cos\theta_1 - 2mv_2\cos\theta_2 = 6mU$$

$$P_{yi} = P_{yf} \quad\Longrightarrow\quad 2mv_2\sin\theta_2 - mv_1\sin\theta_1 = 0$$

Now we note that this is two equations in two unknowns. The unknowns are v_2 and U. We are told to treat v_1, v_3, θ_1 , $theta_2$, and m as known. Solving the second equation for v_2 gives:

$$\underline{\underline{v_2 = \frac{v_1\sin\theta_1}{2\sin\theta_2}}}$$

and substituting this into the first equation and solving for U gives

$$\underline{\underline{U = \frac{v_3}{2} - \frac{v_1}{6}\left(\cos\theta_1 + \frac{\sin\theta_1}{\tan\theta_2}\right)}}$$

Problem 3.

(a) (See exercises 3 and 6 in chapter XIV.) Considering the bug and the rod as a system, there are no external torques, so the angular momentum is constant. The rod is massless, so it has no angular momentum. So, the angular momentum of the bug is constant. Thus $l(t) = l(0)$.

$$\vec{l} = \vec{r} \times \vec{p} = r\hat{r} \times m(v_r\hat{r} + r\omega\hat{\theta}) = mr^2\omega(\hat{r} \times \hat{\theta})$$

$$l(t) = l(0) = \underline{\underline{mD^2\omega_0}}$$

(b) (This is basically quiz 24. The difference is that here we have to find $\omega(t)$ like in the problems mentioned above.) First, we use Newton's second law:

$$\vec{F} = m\vec{a} = m\left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + m\left(2\omega\frac{dr}{dt} + r\alpha\right)\hat{\theta}$$

In the above, we know $r(t) = D + k_1t^2$, so we can find

$$\frac{dr}{dt} = 2k_1t \quad \text{and} \quad \frac{d^2r}{dt^2} = 2k_1$$

Next we need $\omega(t)$. From part (a) we know that the angular momentum is constant, so

$$mD^2\omega_0 = mr^2\omega \quad \Longrightarrow \quad \omega(t) = \frac{D^2}{(D + k_1t)^2}\omega_0$$

The only thing left to find is α

$$\alpha = \frac{d\omega}{dt} = -\frac{4D^2k_1\omega_0t}{(D + k_1t)^3}$$

Substituting all this into the expression above for $\vec{F}$ was not required. It is an ugly mess.

(c) $\vec{\tau} = \vec{r} \times \vec{F}$. Just like in part (a), the $\hat{r}$ part vanishes, so $\underline{\underline{\tau = rF_\theta}}$ where F_θ is given above in part (b).

Problem 4. (For part (a), first, check out problems 2, 3, or 5 or exercise 4 in chapter XIII. For part (b) look at the section in chapter XII called "Calculating Work and Potential Functions." Then look problem 1 in chapter XII.)

(a) For circular motion, r is a constant. Furthermore, there is no reason to think the planet would be speeding up or slowing down in a circular orbit, so ω is constant. Thus, $\vec{a} = -r\omega^2\hat{r}$. This is just the radial acceleration formula that we have used over and over. Using Newton's second law:

$$F_r = ma_r \quad \Longrightarrow \quad -K\frac{m_sm_p}{R^4} = -mR\omega^2 \quad \Longrightarrow \quad \underline{\underline{\omega = \sqrt{\frac{Km_s}{R^5}}}}$$

(b) $$U(2R) - U(R) = -\int_R^{2R} \vec{F} \cdot d\vec{r} = -\int_R^{2R} F_r\,dr(-1) = Km_sm_p\int_R^{2R} r^{-4}dr = \underline{\underline{\frac{7Km_sm_p}{24R^3}}}$$