Physics 218: Mechanics. Final Exam, Secs. 810–812 11 Dec. 2007

Instructor: Dr. George R. Welch, 301D Doherty Building, 845-1571

Print your name **neatly**:

Last name:																	

First name:																	

Sign your name: ______________________________

Please fill in your Student ID number (UIN): _ _ _ – _ _ – _ _ _ _

IMPORTANT

Read these directions carefully:

- There are 8 problems totalling 200 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.
- **Indicate what you are doing!** We cannot give full credit for merely writing down the answer. **Neatness counts!** I will give generous partial credit **if** I can tell that you are on the right track. This means you must be *neat* and organized.
- Each problem with its associated figure is self explanatory. If you *must* ask a question, then come to the front, being as discrete as possible so as not to disturb others.
- Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.
- Do not write below here:

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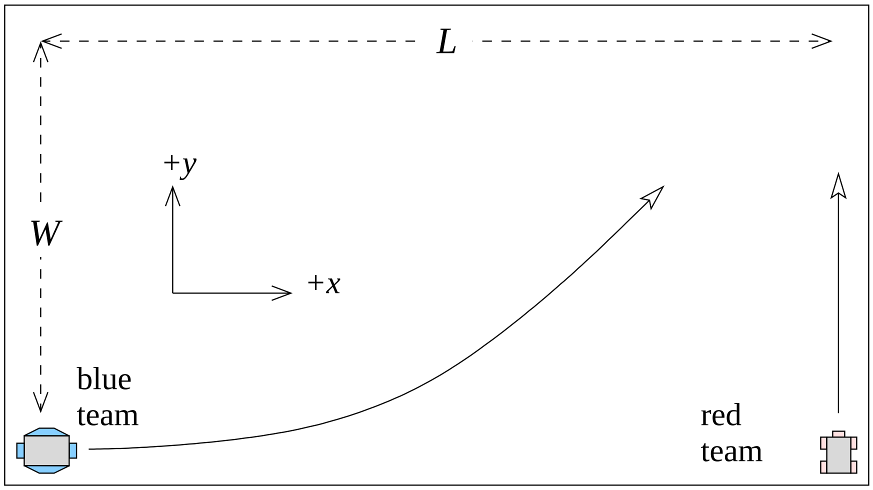
Problem 1. 25 points.

Two teams have built robots that move on a horizontal field of dimensions $L \times W$ as shown. The "red" team just wants their robot to move across the field. The "blue" team wants their robot to catch the red one. We use an x-y coordinate system as shown:



The red team's robot starts at rest, with an acceleration that increases with time according to $\vec{a}_{\text{red}} = c_2 t^2 \,\hat{\jmath}$ where c_2 is a given constant.

The blue team's robot starts with initial velocity $\vec{v}_0 = v_0 \,\hat{\imath}$ toward the red team, and acceleration $\vec{a}_{\text{blue}} = c_1 t \,\hat{\jmath}$ where c_1 is a given constant.

(a) (20 points) If c_1 and c_2 are known, calculate what initial velocity v_0 the blue team should use so that their robot does catch the red team's robot.

(b) (5 points) Using the answer to part (a), calculate the minimum value that c_1 must have so that the blue team's robot catches the red teams's robot before it gets across the field. Express your answer in terms of L, W, and c_2.

Don't forget to be neat.

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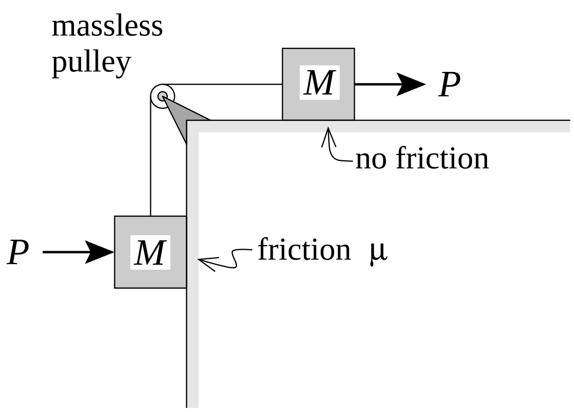
Problem 2. 25 points.

A block of mass M can slide without friction along a horizontal surface. A constant force P pulls the block to the right. On the left, a light strong rope is connected from the block, over a massless pulley, to another identical block of mass M which is suspended by the rope. This second mass is pressed against a rough vertical surface by another constant force P (same magnitude as on the first block.) This arrangement is shown in the Figure.



Note: The two blocks have the same mass M. Do not assign them different masses.

Suppose that the force P is large enough, and that the coefficient of friction μ is small enough, that the block on the horizontal surfaces moves *to the right*

(a) (10 points) Draw **neat** and clear free-body diagrams for each block. Indicate your coordinate system. Neatness will count as a significant part of your grade.

(b) (15 points) Calculate the acceleration of the masses.

You must be **neat.** Neatness helps.

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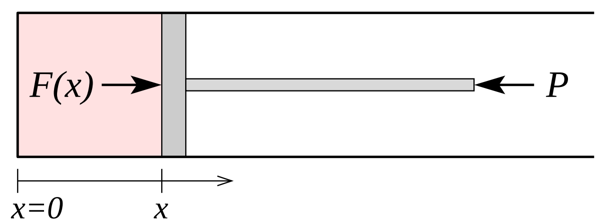
Problem 3. 25 points.

Many engines allow fuel to explode in a confined space, pushing a massive piston along a cylinder. A useful model of this process is called "adiabatic expansion" in which the force on the piston is written as

$$F(x) = \frac{A}{x^\gamma}$$

where x is the distance the gas has expanded relative to zero volume, A is a constant that depends on the energy released, and γ is a property of the gas that is burned.

Suppose that such an expansion is taking place in a cylinder, and that a constant force P pushes back. This is shown here:



Suppose that the fuel is ignited when the position of the piston is x_0 and that the velocity of the piston is 0 at that point. Further, suppose that $F(x_0) > P$ so the piston then initially moves to the right. It continues to move until the constant force P finally causes the rightward motion to cease and to start the piston moving to the left again.

Derive a simple algebraic equation for the maximum position of the piston. Your equation must contain only one unknown, $x_{\rm m}$ which is the maximum position. It can also contain the given quantities: A, γ, P, and x_0. You do not need to solve for $x_{\rm m}$.

Make sure you are being neat. Working neatly will help you get it right.

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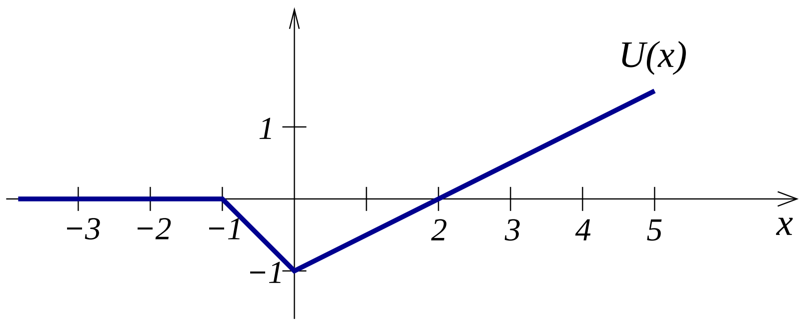
Problem 4. (25 points)

A particle with known mass of exactly 1 kg moves in one dimension under the influence of only conservative forces. The potential energy function for the particle is plotted below:



In the plot, x is measured in meters, and potential energy is measured in joules.

(a) (6 points) For each of the following three regions, indicate if the net force is in the positive or negative direction, and calculate the magnitude of the force in newtons.

(i) $x < -1$ (ii) $-1 < x < 0$ (iii) $x > 0$

(b) (6 points) The particle is at position $x = -3$ meters and has velocity $v = +\sqrt{2}$ meters/second. What will be the maximum position the particle eventually reaches?

(c) (5 points) Where will the particle have maximum velocity?

(d) (8 points) For the initial condition given in part (b), what will be the particle's maximum velocity?

Did you work neatly? Neatness is important.

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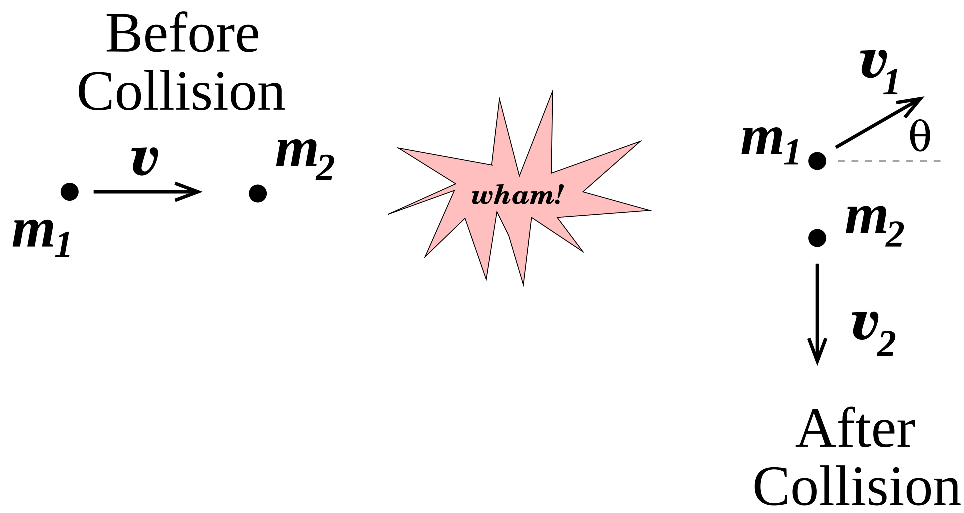
Problem 5. (25 points)

Two balls, with masses m_1 and m_2, move without friction in a horizontal plane. Ball 1 moves in the $+x$-direction with speed v and ball 2 is stationary at the origin. The balls collide inelastically.

After the collision, ball 1 moves at an angle θ from the x-axis, and ball 2 moves in the $-y$-direction.

Calculate the speeds of both balls after the collision in terms of m_1, m_2, v, and θ.



Organize your thoughts, and write clearly. Otherwise I won't give as many points.

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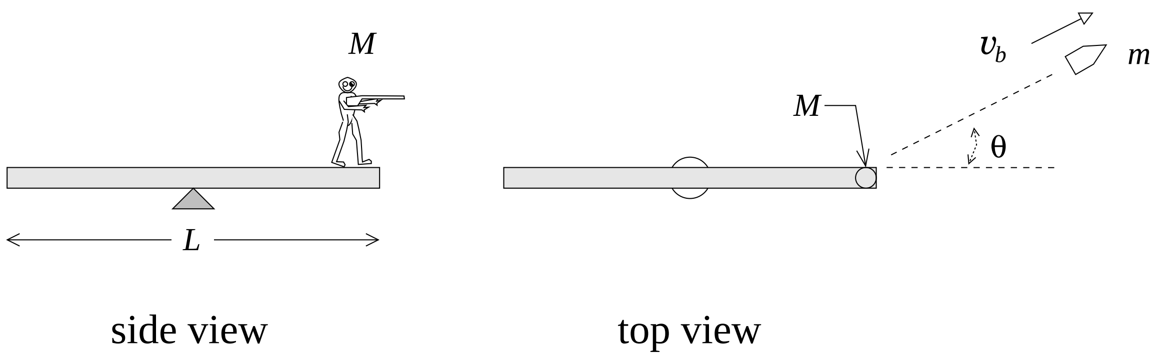
Problem 6. (25 points)

A man of mass M stands on a massless rod which is free to rotate about its center in the horizontal plane. The man has a gun (also considered massless) with one bullet of mass m. Initially the man and rod are stationary.

The man shoots the bullet with velocity v_b horizontally. Find the angular velocity of the man (and rod) as a function of the angle θ which the bullet's velocity vector makes with the rod.



Make sure you are being neat. Working neatly will help you get it right.

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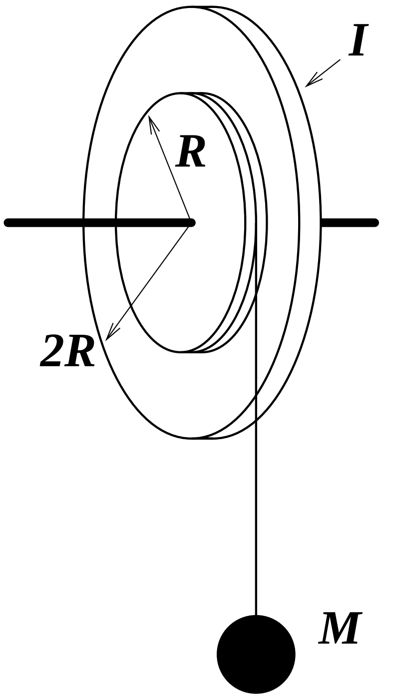
Problem 7. (25 points)

Two metal disks, one with radius R and the other with radius $2R$, are welded together and mounted on a frictionless axis through their common center. A light string is wrapped around the edge of the smaller disk and a block of mass M is suspended from the free end of the string. The moment of inertia of the pulley is I.



(a) (20 points) Calculate the acceleration of the mass M.

(b) (5 points) Same question, but assume that the rope is wrapped around the larger disk.

Note: A similar figure has appeared on several of my old final exams. This is not the same problem. Read the problem.

Always work neatly. Indicate what you are doing to get partial credit.

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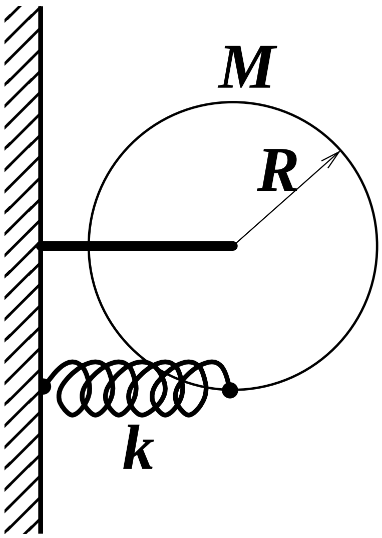
Problem 8. (25 points)

A uniform solid disk of mass M and radius R is free to rotate about its center. Its moment of inertia about its center is $I = MR^2/2$. At the outer edge is attached a spring of force constant k. The other end of the spring is attached to a solid wall. Calculate the period of small oscillations of the disk about its center.



Show your steps and **neatly** indicate what you are doing. There will be no credit for just writing down the answer.

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If you do remove this sheet,
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Potentially useful equations

Calculus:

Derivatives:

$$\text{If } x(t) = C\,t^n \quad \text{then} \quad \frac{dx}{dt} = C\,n\,t^{n-1}$$

Integrals:

$$\int_{t_1}^{t_2} C\,t^n\,dt = C\left[\frac{t_2^{n+1}}{n+1} - \frac{t_1^{n+1}}{n+1}\right]$$

Acceleration in polar coordinates:

$$\vec{a} = \left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(2\omega\frac{dr}{dt} + r\alpha\right)\hat{\theta}$$

where

$$\omega = \frac{d\theta}{dt} \quad \text{and} \quad \alpha = \frac{d\omega}{dt}$$

If $\vec{F}$ is conservative:

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = -[U(\vec{r}_2) - U(\vec{r}_1)]$$

and

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y}$$

Further equations:

$$\vec{L} = \vec{r}\times\vec{p} \qquad \vec{\tau} = \vec{r}\times\vec{F} \qquad I = \sum m_i r_i^2$$

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