Problem 1.
(a) Free-body diagrams:

\[ m_1: \begin{array}{c} +y \\ \downarrow \\ \theta \\ \downarrow N_1 \\ \downarrow m_1 g \end{array} \]

\[ T \]

\[ m_2: \begin{array}{c} +x \\ \uparrow +x \\ m_2 g \end{array} \]

(b)
Block 1: \( \Sigma F_x = m_1 g \sin \theta - T - \mu N = m_1 a \)
\( \Sigma F_y = N - m_1 g \cos \theta = 0 \)
Block 2: \( \Sigma F_x = T - m_2 g = m_2 a \)
Eliminate \( T \):
\[ m_1 g \sin \theta - \mu m_1 g \cos \theta - m_2 g = m_1 a + m_2 a \]
So:
\[ a = \frac{m_1 (\sin \theta - \mu \cos \theta) - m_2}{m_1 + m_2} g \]

Problem 2.
(a)
\[ W_{\text{total}} = K_f - K_i \]
\[ \int_A^{x_f} F(x)dx = 0 - \frac{1}{2} mv_1^2 \]
\[ \int_A^{x_f} \left( -\frac{\alpha}{x^2} - \mu N \right) dx = -\frac{1}{2} mv_1^2 \]
\[ \alpha \left( \frac{1}{x_f} - \frac{1}{A} \right) - \mu N (x_f - A) = -\frac{1}{2} mv_1^2 \]
Solve for \( x_f \).

(b)
\[ \int_A^{x_f} \frac{\alpha}{x^2} dx + \int_A^{x_f} \frac{\alpha}{x^2} dx + \int_A^{x_f} -\mu N dx + \int_A^{x_f} +\mu N dx = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_1^2 \]
First two terms cancel, so:
\[ \int_A^{x_f} -\mu N dx + \int_A^{x_f} +\mu N dx = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_1^2 \]
\[ 2\mu N (A - x_f) = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_1^2 \]
so
\[ \frac{1}{2} mv_f^2 = \frac{1}{2} mv_1^2 + 2\mu N (A - x_f) \]
or
\[ v_f = \sqrt{v_1^2 + 4\mu g (A - x_f)} \]
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Problem 3.

(a) \[ W_{\text{total}} = K_f - K_i \]
\[ \int_0^A F_x(x)\,dx = \frac{1}{2}mv^2(0) - \frac{1}{2}mv_A^2 \]
\[ \int_0^A (-kx + c_1)\,dx = \frac{1}{2}mv^2(0) - 0 \]
\[ -\frac{1}{2}(0^2 - (-A)^2) + c_1(0 - (-A)) = \frac{1}{2}mv^2(0) \]
\[ \frac{1}{2}kA^2 + c_1A = \frac{1}{2}mv^2(0) \]
\[ v(0) = \sqrt{\frac{k}{m}A^2 + \frac{2c_1A}{m}} \]

(b)
\[ \int_0^B c_1\,dx = \frac{1}{2}mv^2(B) - \frac{1}{2}mv^2(0) \]
\[ \frac{1}{2}mv^2(B) = c_1B + \frac{1}{2}mv^2(0) \]
\[ \frac{1}{2}mv^2(B) = c_1B + \frac{1}{2}kA^2 + c_1A \]
\[ v(B) = \sqrt{\frac{k}{m}A^2 + \frac{2c_1(A + B)}{m}} \]

(c)
\[ \int_B^L c_1\,dx + \int_0^H (c_2 - mg)\,dy = \frac{1}{2}mv^2(D) - \frac{1}{2}mv^2(B) \]
\[ c_1(L - B) + c_2H - mgH = \frac{1}{2}mv^2(D) - c_1(A + B) + \frac{1}{2}kA^2 \]
\[ \frac{1}{2}mv^2(D) = \frac{1}{2}kA^2 + c_1(A + L) + (c_2 - mg)H \]
\[ v(D) = \sqrt{\frac{k}{m}A^2 + \frac{2}{m}(c_1(A + B) + (c_2 - mg)H)} \]

Problem 4.

(a) Let’s pick our reference point \( x_1 = +\infty \) and \( U(x_1) = 0 \). So

\[ U(x) = -\int_{x_1}^x F(x)\,dx + U(x_1) = \int_{x_1}^x F(x)\,dx = -\int_{\infty}^x \left( \frac{\alpha}{x^3} - \frac{\beta}{x^2} \right)\,dx = -\alpha \left[ \frac{x^{-2}}{-2} \right]_{\infty}^{x} + \beta \left[ \frac{x^{-1}}{-1} \right]_{\infty}^{x} \]
\[ = \frac{\alpha}{2x^2} - \frac{\beta}{x} \]

Verify:
\[ -\frac{dU}{dx} = \frac{\alpha}{x^3} - \frac{\beta}{x^2} = F(x) \quad \text{Q.E.D.} \]

(b)
\[ E_i = E_f \]
\[ U \left( \frac{\alpha}{2\beta} \right) + \frac{1}{2}mv^2 = U(A) + \frac{1}{2}mv^2(A) \]
\[ \frac{\alpha}{2} \frac{4\beta^2}{\alpha^2} - \beta \frac{2\beta}{\alpha} + \frac{1}{2}mv^2 = \frac{\alpha}{2A^2} - \beta \frac{A}{\alpha} + \frac{1}{2}mv^2(A) \]
\[ 0 + \frac{1}{2}mv^2 = \frac{\alpha}{2A^2} - \beta \frac{A}{\alpha} + \frac{1}{2}mv^2(A) \quad \Rightarrow \text{gives } v(A) \]