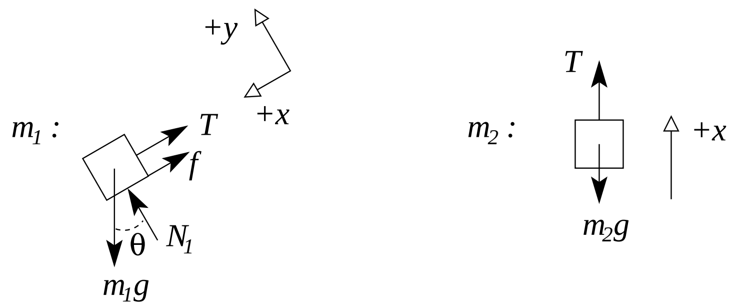
Problem 1.

(a) Free-body diagrams:



(b)

$$\begin{aligned}
\text{Block 1:} \quad & \Sigma F_x = m_1 g \sin\theta - T - \mu N = m_1 a \\
& \Sigma F_y = N - m_1 g \cos\theta = 0 \\
\text{Block 2:} \quad & \Sigma F_x = T - m_2 g = m_2 a
\end{aligned}$$

$$\text{Eliminate } T: \quad m_1 g \sin\theta - \mu m_1 g \cos\theta - m_2 g = m_1 a + m_2 a$$

$$\text{So:} \quad \underline{\underline{a = \frac{m_1(\sin\theta - \mu\cos\theta) - m2}{m_1 + m_2} g}}$$

Problem 2.

(a)

$$\begin{aligned}
W_{\text{total}} &= K_f - K_i \\
\int_A^{x_f} F(x) dx &= 0 - \frac{1}{2} m v_1^2 \\
\int_A^{x_f} \left(-\frac{\alpha}{x^2} - \mu N \right) dx &= -\frac{1}{2} m v_1^2 \\
\alpha \left(\frac{1}{x_f} - \frac{1}{A} \right) - \mu N (x_f - A) &= -\frac{1}{2} m v_1^2 \quad \underline{\underline{\text{Solve for } x_f\,.}}
\end{aligned}$$

(b)

$$\int_A^{x_f} -\frac{\alpha}{x^2} dx + \int_{x_f}^{A} -\frac{\alpha}{x^2} dx + \int_A^{x_f} -\mu N dx + \int_{x_f}^{A} +\mu N dx = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_1^2$$

First two terms cancel, so:

$$\begin{aligned}
\int_A^{x_f} -\mu N dx + \int_{x_f}^{A} +\mu N dx &= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_1^2 \\
2\mu N (A - x_f) &= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_1^2
\end{aligned}$$

so

$$\frac{1}{2} m v_f^2 = \frac{1}{2} m v_1^2 + 2\mu N (A - x_f)$$

or

$$\underline{\underline{v_f = \sqrt{v_1^2 + 4\mu g\,(A - x_f)}}}$$

Problem 3.

(a)

$$\begin{aligned}
W_{\text{total}} &= K_f - K_i \\
\int_0^A F_x(x)dx &= \frac{1}{2}mv^2(0) - \frac{1}{2}mv_A^2 \\
\int_0^A (-kx + c_1)\,dx &= \frac{1}{2}mv^2(0) - 0 \\
-\frac{1}{2}\left(0^2 - (-A)^2\right) + c_1(0 - (-A)) &= \frac{1}{2}mv^2(0) \\
\frac{1}{2}kA^2 + c_1 A &= \frac{1}{2}mv^2(0)
\end{aligned}$$

$$\underline{\underline{v(0) = \sqrt{\frac{k}{m}A^2 + \frac{2c_1 A}{m}}}}$$

(b)

$$\begin{aligned}
\int_0^B c_1 dx &= \frac{1}{2}mv^2(B) - \frac{1}{2}mv^2(0) \\
\frac{1}{2}mv^2(B) &= c_1 B + \frac{1}{2}mv^2(0) \\
\frac{1}{2}mv^2(B) &= c_1 B + \frac{1}{2}kA^2 + c_1 A
\end{aligned}$$

$$\underline{\underline{v(B) = \sqrt{\frac{k}{m}A^2 + \frac{2c_1(A+B)}{m}}}}$$

(c)

$$\begin{aligned}
\int_B^L c_1 dx + \int_0^H (c_2 - mg)dy &= \frac{1}{2}mv^2(D) - \frac{1}{2}mv^2(B) \\
c_1(L - B) + c_2 H - mgH &= \frac{1}{2}mv^2(D) - c_1(A+B) + \frac{1}{2}kA^2 \\
\frac{1}{2}mv^2(D) &= \frac{1}{2}kA^2 + c_1(A+L) + (c_2 - mg)H
\end{aligned}$$

$$\underline{\underline{v(D) = \sqrt{\frac{k}{m}A^2 + \frac{2}{m}\left(c_1(A+B) + (c_2 - mg)H\right)}}}$$

Problem 4.

(a) Let's pick our reference point $x_1 = +\infty$ and $U(x_1) = 0$. So

$$\begin{aligned}
U(x) &= -\int_{x_1}^x F(x)dx + U(x_1) = -\int_\infty^x F(x)dx = -\int_\infty^x \left(\frac{\alpha}{x^3} - \frac{\beta}{x^2}\right)dx = -\alpha\left[\frac{x^{-2}}{-2}\right]_\infty^x + \beta\left[\frac{x^{-1}}{-1}\right]_\infty^x \\
&= \frac{\alpha}{2x^2} - \frac{\beta}{x}
\end{aligned}$$

Verify:

$$-\frac{dU}{dx} = \underline{\underline{\frac{\alpha}{x^3} - \frac{\beta}{x^2} = F(x)}} \quad \text{Q.E.D.}$$

(b)

$$\begin{aligned}
E_i &= E_f \\
U\left(\frac{\alpha}{2\beta}\right) + \frac{1}{2}mv_1^2 &= U(A) + \frac{1}{2}mv^2(A) \\
\frac{\alpha}{2}\frac{4\beta^2}{\alpha^2} - \beta\frac{2\beta}{\alpha} + \frac{1}{2}mv_1^2 &= \frac{\alpha}{2A^2} - \frac{\beta}{A} + \frac{1}{2}mv^2(A) \\
0 + \frac{1}{2}mv_1^2 &= \frac{\alpha}{2A^2} - \frac{\beta}{A} + \frac{1}{2}mv^2(A) \qquad \Rightarrow \underline{\underline{\text{gives } v(A)}}
\end{aligned}$$