Problem 1a. (This is the whole point of chapter 12 in the book. I did it in class. I told you it would be on the exam.)

$$\hat{r} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y} \qquad \hat{\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$$

$$\frac{d\hat{r}}{d\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y} = \hat{\theta} \qquad \frac{d\hat{\theta}}{d\theta} = -\cos\theta\,\hat{x} - \sin\theta\,\hat{y} = -\hat{r}$$

$$\frac{d\hat{r}}{dt} = \frac{d\hat{r}}{d\theta}\frac{d\theta}{dt} = \hat{\theta}\omega \qquad \frac{d\hat{\theta}}{dt} = \frac{d\hat{\theta}}{d\theta}\frac{d\theta}{dt} = -\hat{r}\omega$$

$$\begin{aligned}\vec{v} &= \frac{d\vec{r}}{dt} = \frac{d}{dt}r\,\hat{r}\\ &= \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt}\\ &= \frac{dr}{dt}\hat{r} + r\omega\,\hat{\theta}\end{aligned}$$

$$\begin{aligned}\vec{a} &= \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{dr}{dt}\hat{r} + r\omega\,\hat{\theta}\right)\\ &= \frac{d^2r}{dt^2}\hat{r} + \frac{dr}{dt}\frac{d\hat{r}}{dt} + \frac{dr}{dt}\omega\,\hat{\theta} + r\frac{d\omega}{dt}\hat{\theta} + r\omega\frac{d\hat{\theta}}{dt}\\ &= \frac{d^2r}{dt^2}\hat{r} + \frac{dr}{dt}\omega\,\hat{\theta} + \frac{dr}{dt}\omega\,\hat{\theta} + r\alpha\,\hat{\theta} - r\omega^2\,\hat{r}\\ &= \underline{\underline{\left(\frac{d^2r}{dt^2} - r\omega^2\right)\hat{r} + \left(2\omega\frac{dr}{dt} + r\alpha\right)\hat{\theta}}}\end{aligned}$$

Problem 1b. (Here, we just wanted to make sure you knew what the symbols meant. Sort of like with the cockroach quiz.)

$$r = kt^2 \quad \text{so} \quad \frac{dr}{dt} = 2kt \quad \text{and} \quad \frac{d^2r}{dt^2} = 2k \qquad \theta = bt^2 \quad \text{so} \quad \frac{d\theta}{dt} = 2bt \quad \text{and} \quad \frac{d^2\theta}{dt^2} = 2b$$

$$\vec{F} = m\vec{a} = \left(2k - (kt^2)(2bt)^2\right)\hat{r} + \left(2(2bt)(2kt) + (2kt^2)(2b)\right)\hat{\theta} = \underline{\underline{\left(2k - 4kb^2t^4\right)\hat{r} + \left(10kbt^2\right)\hat{\theta}}}$$

Problem 2. (Based on chapter 10, exercise 6, or "A Famous Problem" on page 195.)

(a) (18 points) For the collision, we will use conservation of linear momentum: $P_i = P_f$. However, first we must find out how fast the ball is moving when it hits the block. That is easy, since it is swinging down under gravity.

$$E_i = E_f \quad \Rightarrow \quad m_b g h = \frac{1}{2}m_b v_b^2$$

where h is the starting height $h = S - S\cos\theta_0$ and v_b is the speed of the ball just *before* the collsion. Solving:

$$v_b = \sqrt{2gS\left(1 - \cos\theta_0\right)}$$

Now for the (totally inelastic) collision,

$$P_i = P_f \quad \Rightarrow \quad m_b v_b = (m_b + m_i)V$$

where V is the speed of the combined ball and block *just after* the collision. Solving

$$V = \frac{m_b}{m_b + m + i}\sqrt{2gS\left(1 - \cos\theta_0\right)}$$

Now, we find out how far the combined ball and block go up:

$$\frac{1}{2}(m_b + m_i)V^2 = (m_b + m_i)gH$$

Solving:

$$\underline{\underline{H = \frac{1}{2}(m_b + m_i)S\left(1 - \cos\theta_0\right)}}$$

(b) (7 points) It is the same until the ball hits the block. That is, v_b is the same as above. However now the collision is more complicated:

$$P_i = P_f \quad \Rightarrow \quad m_b v_b = m_b V_b + m_i V_i$$

where V_b and V_i are the speeds of the block and ball just after the collision. We can't solve for them with that equation alone, so we make use of the fact that the collision is elastic:

$$P_i = P_f \quad \Rightarrow \quad \frac{1}{2}m_b v_b^2 = \frac{1}{2}m_b V_b^2 + \frac{1}{2}m_i V_i^2$$

and then we could solve for V_b and V_i. We would then find the heights as above.

Problem 3.

(a) (8 points) (Based on angular kinematics. Chapter 13, exercise 1, or exercise 5 in chapter 15.) We want to reach an angular velocity of ω_0 in time t_0 so $\alpha = \omega_0/t_0$. Suppose we apply our force perpendicularly, then

$$\tau = I\alpha \quad \Rightarrow \quad FS = I\frac{\omega_0}{t_0}$$

But $I = 2mS^2$ so

$$FS = 2mS^2\frac{\omega_0}{t_0} \quad \Rightarrow \quad \underline{\underline{F = 2mS\frac{\omega_0}{t_0}}}$$

(b) (12 points) (See page 287-288, and problem 4 in chapter 15.) We use conservation of the vertical component of the angular momentum. We recall $L_z = I\omega$ so

$$L_i = L_f \quad \Rightarrow \quad 2mS^2\omega_0 = 2m\left(\frac{S}{4}\right)^2\omega \quad \Rightarrow \quad \underline{\underline{\omega = 16\omega_0}}$$

(c) (5 points) Just the same again:

$$L_i = L_f \quad \Rightarrow \quad \left(I_{\text{man}} + 2mS^2\right)\omega_0 = \left(I_{\text{man}} + 2m\left(\frac{S}{4}\right)^2\right)\omega \quad \Rightarrow \quad \underline{\underline{\omega = \frac{(I_{\text{man}} + 2mS^2)}{(I_{\text{man}} + mS^2/8)}\omega_0}}$$

Problem 4. See problem 2, chapter 15.

(a) (10 points) $\vec{\tau} = \vec{r} \times \vec{F} = \left(\frac{S}{2}\hat{r}\right) \times (-mg\hat{z}) = \frac{mgS}{2}(-\hat{r} \times \hat{z}) = \underline{\underline{\frac{mgS}{2}\,\hat{\theta}}}$ (into page)

(b) (5 points) $\tau = I\alpha \quad \Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{mgS}{2I} \qquad a = S\alpha = \underline{\underline{\frac{mgS^2}{2I}}}$

(c) (7 points) $\vec{\tau} = \vec{r} \times \vec{F} \quad \Rightarrow \quad \underline{\underline{\tau = 0}}$ because $\vec{F}$ points through the origin (perpendicular distance = 0).

(d) (3 points) This is motion on a circle, so $\vec{a} = -r\omega^2\,\hat{r} + r\alpha\,\hat{\theta}$. But from part (c) we know $\tau = 0$ so $\alpha = 0$. Thus, $\underline{\underline{a = S\omega_f^2}}$.