Problem 1
(a) Free Body diagrams:

\[ +x \quad N_1 \quad T \quad N_2 \quad T \]

\[ P \quad f_1 \quad Mg \quad f_2 \quad Mg \]

(b,c) We use Newton’s second law. First, we write the +x component for each of the two blocks:

\[ P - T - f_1 = Ma \quad \text{and} \quad T - f_2 - Mg = Ma \]

Next we recall that for kinetic friction, \( f = \mu N \), so we need the Normal forces. Using the y component of Newton’s law, we see \( N_1 = Mg \) and \( N_2 = P \). So, \( f_1 = \mu Mg \) and \( f_2 = \mu P \). Substituting this into the above gives:

\[ P - T - \mu Mg = Ma \quad \text{and} \quad T - \mu P - Mg = Ma \]

This is two equation in two unknowns: \( a \) and \( T \). Solving these gives:

\[ a = \frac{P(1 - \mu) - Mg(1 + \mu)}{2M} \quad \text{and} \quad T = \frac{P(1 + \mu) + Mg(1 - \mu)}{2} \]

Problem 2
(a) We use the work energy theorem. We look at the process starting when the spring is at rest \( x = 0 \) and then reaching its maximum distance \( D \). At the point \( x = D \) we realize the velocity is again zero.

\[ W_{\text{total}} = K_f - K_i \]

\[ \int_{x_i}^{x_f} F(x)dx = \frac{1}{2} Mv_f^2 - \frac{1}{2} Mv_i^2 \]

\[ \int_0^D (P - kx - T_1)dx = 0 - 0 \]

\[ (P - T_1)D - \frac{1}{2} kD^2 = 0 \]

\[ P - T_1 = \frac{1}{2} kD \]

\[ T_1 = P - \frac{1}{2} kD \]

(b) We use Newton’s second law:

\[ F = Ma \]

\[ P - kD - T_1 = Ma \]

\[ P - kD - \left( P - \frac{1}{2} kD \right) = Ma \]

\[ -\frac{1}{2} kD = Ma \]

\[ a = \frac{kD}{2M} \]
Problem 3

(a) \[ F(x) = -\frac{dU}{dx} = -\frac{d}{dx} \alpha(x - x_0)^2 = -2\alpha(x - x_0) \]

(b) \[ E = K + U = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}m\left(\frac{3\alpha}{m}x_0^2\right) + 0 = \frac{3}{2}\alpha x_0^2 \]

(c) \[
E = \frac{1}{2}mv^2 + U(x) \\
\frac{3}{2}\alpha x_0^2 = \frac{1}{2}mv^2 + \alpha(2x_0 - x_0)^2 \\
\frac{3}{2}\alpha x_0^2 - \alpha x_0^2 = \frac{1}{2}mv^2 \\
\frac{1}{2}\alpha x_0^2 = \frac{1}{2}mv^2 \\
v = \sqrt{\frac{\alpha}{m}x_0}
\]

Problem 4

Let \( V \) be the velocity of the second block after the collision:

(a) \[ Mv_0 = 3MV \quad \rightarrow \quad V = \frac{v_0}{3} \]

(b) \[ Mv_0 = -M\frac{v_0}{3} + 2MV \quad \rightarrow \quad V = \frac{2v_0}{3} \]

\[
\frac{1}{2}3MV^2 = \frac{1}{2}kx^2 \\
\frac{1}{2}3M\left(\frac{v_0}{3}\right)^2 = \frac{1}{2}kx^2 \\
x = \sqrt{\frac{M}{3k}v_0}
\]

\[
\frac{1}{2}2MV^2 = \frac{1}{2}kx^2 \\
\frac{1}{2}2M\left(\frac{2v_0}{3}\right)^2 = \frac{1}{2}kx^2 \\
x = \frac{2}{3}\sqrt{\frac{2M}{k}v_0}
\]

(c) The answer to part (b) is much larger. But it is easy to see the collision in part (b) is elastic: Before the collision the relative velocity is \( v_0 \). After the collision the relative velocity is \( 2v_0/3 - (-v_0/3) = v_0 \). So, the relative velocity is the same after the collision, so the collision in part (b) is elastic.