Problem 1.  (I announced in class, on several occasions, that this exact problem would be on the exam.)

\[ \hat{r} = \cos \theta \hat{x} + \sin \theta \hat{y} \]
\[ \frac{d\hat{r}}{d\theta} = -\sin \theta \hat{x} + \cos \theta \hat{y} = \hat{\theta} \]
\[ \frac{d\hat{r}}{dt} = \frac{d\hat{r}}{d\theta} \frac{d\theta}{dt} = \hat{\theta} \omega \]
\[ \hat{\theta} = -\sin \theta \hat{x} + \cos \theta \hat{y} \]
\[ \frac{d\hat{\theta}}{d\theta} = -\cos \theta \hat{x} - \sin \theta \hat{y} = -\hat{r} \]

\[ \vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt} r \hat{r} \]
\[ \vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} \left( \frac{dr}{dt} \hat{r} + r \omega \hat{\theta} \right) \]

Problem 2  (Pretty much identical to 9.87.)

(a) \[ I = \frac{1}{2} MR^2 + \frac{1}{2} (2M)(2R)^2 = \left( \frac{1}{2} + 4 \right) MR^2 = \frac{9}{2} MR^2 \]

(b) \[ E_i = E_f \]
\[ Mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2 \]
\[ = \frac{9}{2} MR^2 \left( \frac{v}{R} \right)^2 + \frac{1}{2} M v^2 \]
\[ = \left( \frac{9}{4} + \frac{1}{2} \right) M v^2 = \frac{11}{4} M v^2 \]
\[ v = \sqrt{\frac{4gh}{11}} \]

(c) \[ E_i = E_f \]
\[ Mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2 \]
\[ = \frac{9}{2} MR^2 \left( \frac{v}{2R} \right)^2 + \frac{1}{2} M v^2 \]
\[ = \left( \frac{9}{16} + \frac{1}{2} \right) M v^2 = \frac{17}{16} M v^2 \]
\[ v = \sqrt{\frac{16gh}{17}} \]
Problem 3 (This was kind of a combination of 10.70 and 10.87.)

Free-body diagrams looks like this:

For the pulley: \( \Sigma \tau = I \alpha \implies TR = \frac{MR^2}{2} \frac{a}{R} \implies T = \frac{Ma}{2} \)

For the disk: \( \Sigma \tau = I \alpha \implies fR = \frac{MR^2}{2} \frac{a}{R} \implies f = \frac{Ma}{2} \)

And also: \( \Sigma F_x = Mg \sin \theta - f - T = Ma \)

Thus: \( Mg \sin \theta - \frac{Ma}{2} - \frac{Ma}{2} = Ma \implies a = \frac{g \sin \theta}{2} \)

Problem 4 (This is basically the same as 10.95.)

This is an inelastic collision, so we cannot use conservation of energy – have to use conservation of momentum. Because the rod is going to rotate, we’ll conservation of angular momentum: \( L_i = L_f \).

After the collision, rotation is about the hinge at the top, so we will use that as our axis and origin.

\[ L_i = |\vec{p} \times \vec{r}| = mvr \sin \theta = mv \frac{2}{3} L \]

\[ L_f = I \omega = (I_{rod} + I_{putty}) \omega = \left( I_{rod} + m \left( \frac{2}{3} L \right)^2 \right) \omega = \left( \frac{ML^2}{12} + M \left( \frac{L}{2} \right)^2 + m \left( \frac{2}{3} L \right)^2 \right) \omega \]

and so: \( \omega = \frac{mv \frac{2}{3} L}{\left( \frac{ML^2}{12} + M \left( \frac{L}{2} \right)^2 + m \left( \frac{2}{3} L \right)^2 \right)} = \frac{6m}{(3M + 4m)} \frac{v}{L} \)

Problem 5. (This was a simplified version of problem 11.14.)

Free-body diagrams looks like this:

(a) Look at the torques about point A:

\( \Sigma \tau_A = 0 \implies T \sin \theta \frac{L}{2} - Mg L = 0 \implies T = \frac{2Mg}{\sin \theta} \)

(b) Look at the Forces:

\( \Sigma F_x = R_x - T \cos \theta = 0 \implies R_x = T \cos \theta = \frac{2Mg}{\tan \theta} \)

\( \Sigma F_y = R_y + T \sin \theta - Mg = 0 \implies R_y = Mg - 2Mg = -Mg \implies |R_y| = Mg \) and so it is pointing down.