Problem 1. This was a simplified version of example 21-8 from the book.

The positive charge produces a field \( \vec{E}_1 \) and the negative charge produces a field \( \vec{E}_2 \) as shown:

\[
\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{2\sqrt{2}Q}{4\pi \varepsilon_0 (\sqrt{2}a)^2} (\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j}) + \frac{-Q}{4\pi \varepsilon_0 a^2} \hat{j} = \frac{2\sqrt{2}Q}{4\pi \varepsilon_0 2a^2} \sqrt{2} \hat{i} + \frac{2\sqrt{2}Q}{4\pi \varepsilon_0 2a^2} \frac{1}{\sqrt{2}} \hat{j} - \frac{Q}{4\pi \varepsilon_0 a^2} \hat{j} = \frac{Q}{4\pi \varepsilon_0 a^2} \hat{i}
\]

Problem 2. This was an exact copy of homework problem 22-28.

(a) Volume of the cylinder is length times cross-sectional area, so charge per unit volume is charge per unit length per unit area.

(b) Gauss’ Law:

\[
\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \quad E 2\pi rl = \frac{\lambda l}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{\lambda}{2\pi \varepsilon_0 r}
\]

(c) Again Gauss’ Law, but now \( Q_{\text{in}} = \rho \pi r^2 l = \lambda \frac{r^2}{R^2} l \) so \( E = \frac{\lambda r}{2\pi \varepsilon_0 R^2} \).

Problem 3.

(a) This was an exact copy of homework problem 21-48.

Use \( E = \int_{\text{charge}} \frac{dq}{4\pi \varepsilon_0 r^2} \). We’ll take the x axis along the rod, with the origin at the end closes to P. Then, \( dq = \lambda dx \) where \( \lambda = Q/L \), and the distance from point P to some position \( x \) is \( x + a \). So,

\[
E = \int_0^L \frac{\lambda dx}{4\pi \varepsilon_0 (x + a)^2} = \frac{\lambda}{4\pi \varepsilon_0} \int_a^{a+L} \frac{du}{u^2} = \frac{\lambda}{4\pi \varepsilon_0} \left[ \frac{1}{a} - \frac{1}{a + L} \right] = \frac{Q}{4\pi \varepsilon_0 a(a + L)}
\]
(3b) This was based on homework problem 23-32, but I used a much simpler geometry here.

Use \( V = \int_{\text{charge}} \frac{dq}{4\pi\epsilon_0 r} \). We’ll take the same axis and differential charge as in part (a), so,

\[
E = \int_{0}^{L} \frac{\lambda dx}{4\pi\epsilon_0 (x + a)}
= \frac{\lambda}{4\pi\epsilon_0} \int_{a}^{a+L} \frac{du}{u}
= \frac{\lambda}{4\pi\epsilon_0} \ln \left( \frac{a + L}{a} \right)
= \frac{Q}{4\pi\epsilon_0 L} \ln \left( \frac{a + L}{a} \right)
\]

Problem 4. In Section 21-2, you integrate \( E \) to get \( V \). This is illustrated in example 23-3 with a constant field and 23-4 with a coulomb field. You also did this integral in homework problem 23-17. I picked an especially easy electric field to integrate to make the problem simple. By the way, the field comes from homework problem 22-36, so no surprises.

\[
V = -\int_{x_0}^{x} E(x) \, dx
= -\int_{0}^{x} \frac{\rho x}{\epsilon_0} \, dx
= -\frac{\rho}{2\epsilon_0} x^2
\]

Problem 5. Based on homework 23-70 and 23-50, but the geometry here is easier than either of those.

Add interaction energies. First bring in charge \(-Q\). Then bring in a \(+2Q\). Then finally the other \(2Q\).

\[
U = 0 + 2Q \left( \frac{-Q}{4\pi\epsilon_0 a} \right) + 2Q \left( \frac{-Q}{4\pi\epsilon_0 a} + \frac{2Q}{4\pi\epsilon_0 (2a)} \right)
= \frac{Q^2}{4\pi\epsilon_0 a} (-2 - 2 + 2)
= -\frac{Q^2}{2\pi\epsilon_0 a}
\]