Print your full name: ________________________________

Sign your name: ________________________________

Please fill in your Student ID number: _______ _______ _______

IMPORTANT

Read these directions carefully:

• There are 4 problems totalling 100 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.

• Indicate what you are doing! We cannot give full credit for merely writing down the answer. Neatness counts! I will give generous partial credit if I can tell that you are on the right track. This means you must be neat and organized.

• Each problem with its associated figure is self explanatory. If you must ask a question, then come to the front, being as discrete as possible so as not to disturb others.

• Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.
A particle moves in 1 dimension, and experiences a single conservative force given by

\[ F = -kx + bx^2 \]

where \( k \) and \( b \) are both given positive constants.

(a) What are the units of the ratio \( k/b \)?

(b) Calculate two positions (values of \( x \)) where the force is equal to zero.

(c) Derive the functional form of the potential energy of the particle, assuming that \( U(x = 0) = 0 \). Draw a box around your answer.

The function that you should get is plotted below. This function has a local minimum at \( x = 0 \) with \( U(0) = 0 \) and a local maximum at \( x = +k/b \) with \( U(k/b) = k^3/(6b^2) \).

Suppose that the particle is released from rest at \( x = -k/b \).

(d) What will be the velocity of the particle when it reaches \( x = 0 \)?

(e) What will be the velocity of the particle when it reaches \( x = +k/b \)?
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Physics 218: Mechanics, Exam 3

Problem 2. 25 points.

A small block of mass $M$ slides without friction along a horizontal plane. The velocity of the block is $v_0$. The block collides with a heavier block of mass $4M$ which is initially at rest. The small block bounces off the heavier one, and rebounds backwards with a velocity $v_0/3$, while the heavy block is knocked into a spring of force constant $k$. The heavy block also slides without friction. What will be the maximum compression of the spring $S$?

Before:

$$v_0$$

$M$ $v=0$ $4M$ $k$

After:

$$v_0/3$$

$M$ $4M$ $S$
Physics 218: Mechanics, Exam 3

Problem 3. 25 points.

You are given expressions for the polar unit vectors in terms of the cartesian unit vectors:

\[ \hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j} \]
\[ \hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j} \]

Start with the the position vector \( \vec{r} = r \hat{r} \) and calculate the acceleration vector \( \vec{a} \) expressed in polar coordinates.

You may need the derivatives:

\[ \frac{d}{d\theta} \cos \theta = -\sin \theta \quad \text{and} \quad \frac{d}{d\theta} \sin \theta = \cos \theta . \]

Show all the steps in the derivation — do not use any other results that have been shown in class. Use the back if necessary.
Print your name: _____________________________________________

Physics 218: Mechanics, Exam 3

Problem 4. 25 points.

A uniform solid cylinder of mass $M$ and radius $R$ rolls without slipping on a horizontal surface. The cylinder has a frictionless axle which is pulled by a light rope. The rope is wrapped around a massless frictionless pulley and connected to a block, also of mass $M$. The block is released from rest. Calculate the acceleration of the block.

The moment of inertia of the wheel about its center is $I = MR^2/2$. If you substitute this value for $I$ in your expressions, it makes the algebra easier.

DO NOT use two values such as $M_1$ and $M_2$ for the two masses. Instead, both masses have mass $M$. This makes the problem easier.