Print your full name: ______________________________________________

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IMPORTANT

Read these directions carefully:

• There are 5 problems totalling 100 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.

• Indicate what you are doing! We cannot give full credit for merely writing down the answer. Neatness counts! I will give generous partial credit if I can tell that you are on the right track. This means you must be neat and organized.

• Each problem with its associated figure is self explanatory. If you must ask a question, then come to the front, being as discrete as possible so as not to disturb others.

• Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.
Physics 218: Mechanics, Exam 3

Problem 1. 25 points.

A particle of mass \( m \) moves in one dimension under the influence of only conservative forces. The potential energy of the particle may be written as

\[
U(x) = \begin{cases} 
0 & x \leq 0 \\
U_0 \left( 3 \left( \frac{x}{b} \right)^2 - 2 \left( \frac{x}{b} \right)^3 \right) & 0 \leq x \leq b \\
U_0 & x \geq b
\end{cases}
\]

where \( U_0 \) and \( b \) are given positive constants. For your convenience, this function is plotted here:

![Potential Energy Plot]

The particle is initially on the left side, but moving to the right. That is, \( x_0 < 0 \) and \( v_0 > 0 \). Thus, the particle will move to the right and continue some distance past \( x = 0 \).

Calculate the velocity and acceleration of the particle at the instant it reaches the point \( x = b/2 \) for each of the following two cases:

(a) \( v_0 = \sqrt{\frac{3U_0}{m}} \)

(b) \( v_0 = \sqrt{\frac{U_0}{m}} \)

Work neatly.
Problem 2. 15 points.

A heavy particle of mass $5M$ is moving in 1 dimension with velocity $v_0$. The particle collides with a smaller particle of mass $M$ that is stationary. The heavier particle is slowed by the collision to a speed that is $2/3$ its initial speed.

Calculate the speed of the smaller particle after the collision.

Is this collision elastic? (Explain.)
Two particles with identical mass $m$ are moving in 2 dimensions. There is no friction, gravity, or any external forces. The particles have different speeds $v_1$ and $v_2$ and their velocities make the same angle $\theta$ from the x-axis as shown.

After the collision, the particle stick together, and move with velocity $v$ at an angle $\phi$ from the x-axis.

Calculate the initial velocities $v_1$ and $v_2$ in terms of $v$ and the angles $\theta$ and $\phi$. **Be neat.** Neatness help.
Problem 4. 15 points.

You are given expressions for the polar unit vectors in terms of the Cartesian unit vectors:

\[ \hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j} \]
\[ \hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j} \]

Start with the the position vector \( \mathbf{r} = r \hat{r} \) and calculate the acceleration vector \( \mathbf{a} \) expressed in polar coordinates.

You may need the derivatives:

\[ \frac{d}{d\theta} \cos \theta = -\sin \theta \text{ and } \frac{d}{d\theta} \sin \theta = \cos \theta . \]

Show all the steps in the derivation — do not use any other results that have been shown in class.

Don’t forget to be neat.
A large flywheel of mass $M$ and radius $R$ is mounted so it can spin freely about its axis. A light strong rope is wrapped around the flywheel, and a block of the same mass $M$ is attached to the rope. When the block is released, the flywheel starts to spin. Treat the flywheel as a uniform solid cylinder with moment of inertia $I = MR^2/2$.

Calculate the magnitude of the acceleration of any point on the outside edge of the flywheel after the block has fallen through a distance equal to one-third the radius of the wheel. That is, after the block has dropped $R/3$.

*Hint:* Acceleration is a vector. In this case it has two components: the radial component depends on the angular velocity of the wheel, and the tangential component can be related to the acceleration of the block.

*Hint²:* Both the block and the flywheel have the same mass $M$. Do not assign different masses to the two.
You may remove this sheet.

If you do remove this sheet, **DO NOT TURN IT IN!**

Potentially useful equations

Calculus:

Derivatives:

If \( x(t) = C t^n \) then \( \frac{dx}{dt} = C n t^{n-1} \)

Integrals:

\[
\int_{t_1}^{t_2} C t^n dt = C \left[ \frac{t_2^{n+1}}{n+1} - \frac{t_1^{n+1}}{n+1} \right]
\]

If you do remove this sheet, **DO NOT TURN IT IN!**