Instructor: Dr. George R. Welch, 415 Engineering-Physics, 845-7737

Print your name **neatly**:

Last name: 

First name: 

Sign your name: ________________________________

Please fill in your Student ID number (UIN): __ __ __ __ __ __ __ __ __ __

IMPORTANT

Read these directions carefully:

• There are 6 problems totalling 100 points. Check your exam to make sure you have all the pages. Work each problem on the page the problem is on. You may use the back. If you need extra pages, I have plenty up front.

• **Indicate what you are doing!** We cannot give full credit for merely writing down the answer. **Neatness counts!** I will give generous partial credit if I can tell that you are on the right track. This means you must be neat and organized.

• Each problem with its associated figure is self explanatory. If you must ask a question, then come to the front, being as discrete as possible so as not to disturb others.

• Put your name on each page it is asked for. You will lose credit if you fail to print your name on each page it is asked for.
Problem 1. 10 points.

Consider three vectors $\mathbf{A}$ and $\mathbf{B}$ and $\mathbf{C}$ as shown. You are given the following:

- The length of vector $\mathbf{A}$ is 5 units and the angle between vector $\mathbf{A}$ and the $x$-axis is $\theta = 53.1^\circ$. (Note — no calculator needed: $\cos \theta = 0.6$ and $\sin \theta = 0.8$.)
- The length of vector $\mathbf{B}$ is 3 units and it points along the $-x$-axis.
- Vector $\mathbf{C}$ has $x$ component $C_x = 2$ units and $y$ component $C_y = 4$ units as shown in the figure.

Calculate the $x$- and $y$- components of the following three vectors, and state which one is longest:

1. $\mathbf{A} + \mathbf{B} + \mathbf{C}$
2. $\mathbf{A} - \mathbf{B} + \mathbf{C}$
3. $\mathbf{A} + \mathbf{B} - \mathbf{C}$

If you work **neatly** I will find more partial credit for you!
The velocity of a particle as a function of time is given by

\[ v(t) = v_0 + kt^2 - \beta t \]

where \( v_0, k, \) and \( \beta \) are positive constants. Suppose the position of the particle at \( t = 0 \) is given to be \( x_0 \).

(a) (8 points) Calculate the acceleration of the particle as a function of time.

(b) (8 points) Calculate the position of the particle as a function of time.

(c) (4 points) Will this particle ever be in equilibrium. If so, when? (That is, calculate the value of \( t \).)

Express all your answers only in terms of \( x_0, v_0, k, \beta, \) and numerical factors.
Problem 3. 25 points.

The football team at the Texas Atomic and Molecular University was recently embarrassed when an opponent ran a very clever route during a play. Unfortunately, no video records of the play were kept. However, a technician at the field was testing radar guns for the baseball team at the time, and he recorded both the $x$- and $y$-components of the opposing player’s velocity.

As a consulting engineer, your job is to analyze the play for the coach, whose job is on the line. You model the play in the following way:

- Choose $t = 0$ to correspond to the start of the play.
- Choose the $x$-axis to be along the line of scrimmage, with the origin at the player in question.
- Choose the $y$-axis to be in the forward direction along the field, with the origin at the player in question.

These choices are shown schematically in the following figure:

In this case, the velocity of the opposing player is well modeled by the following equation:

$$\vec{v}(t) = At\hat{i} + (B - Ct^2)\hat{j}$$

where $A$, $B$, and $C$ are positive constants.

(a) (7 points) Derive an equation for the acceleration vector of the player as a function of time.

(b) (8 points) Derive an equation for the position vector of the player as a function of time.

(c) (5 points) Calculate how far forward the player gets. (That is, calculate the maximum value of $y$ obtained by the player.)

(d) (5 points) Calculate the $y$-component of the speed of the player when he returns to the $x$-axis.

You may use the next page to complete this problem.

NOTE: Express your answer’s only in terms of $A$, $B$, $C$, and numerical factors.
Problem 4. (20 points)

A stunt-person drives a motorcycle off a cliff. The speed of the motorcycle at the moment it leaves the cliff is $v_0$ and the height of the cliff is $H$. At the exact instant that the motorcycle leaves the cliff, a truck with soft padding in its bed leaves the bottom of the cliff to catch the stunt-person safely. Assume that the truck has constant acceleration $a$, and treat both the truck and stunt-person as point objects. Ignore air resistance.

(a) For how long is the stunt-person in the air?

(b) How far from the base of the cliff does the stunt-person land?

(c) What must be the acceleration of the truck to safely catch the stunt-person?

Note: Express all your answers only in terms of $H$, $v_0$, $a$, $g$, and numerical factors.

You need to work Neatly! Don’t forget to be neat.
Problem 5. (10 points)

Two blocks, of mass $M_1$ and $M_2$, slide without friction on a plane that is inclined at an angle $\theta$ above the horizontal. The blocks are connected by a light strong taut rope. A constant force $P$ is applied to the upper block ($M_1$) in a direction parallel to the plane as shown in the Figure.

Draw free body diagrams for each block. Label each diagram with $x$- and $y$-axes that would be convenient for analyzing the dynamics.

You must be neat. Neatness will count as part of the grade on this problem.
Problem 6. (15 points)

Refer to the previous problem. Calculate the tension in the rope, and the acceleration of the blocks.

For this problem, assume both blocks have the same mass $M$, that is, $M_1 = M_2 = M$. Do not keep $M_1$ and $M_2$ separately in your calculations; use $M$.

Present your work neatly and clearly.
Potentially useful equations

Calculus:

Derivatives:

\[ x(t) = Ct^n \quad \text{then} \quad \frac{dx}{dt} = Cn t^{n-1} \]

Integrals:

\[ \int_{t_1}^{t_2} C t^n \, dt = C \left[ \frac{t_2^{n+1}}{n+1} - \frac{t_1^{n+1}}{n+1} \right] \]